LEEDS PHILOSOPHICAL AND LITERARY SOCIETY.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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A. A. COMMON, LL.D., F.R.S., President, in the Chair.

William Johnston, Saint Nicholas House, Westgate Street, Gloucester;

George Edward Lumsden, Assistant Secretary of the Province of Ontario, 57 Elm Avenue, Toronto, Canada;

Major Percy Alexander MacMahon, R.E., F.R.S., Artillery College, Woolwich, and 52 Shaftesbury Avenue, W.;

Louis C. H. Maston, San Francisco, California, U.S.A.;

Beauchamp Prideaux Selby, J.P., Pawston, Cornhill-on-Tweed, Northumberland;

J. E. de Villiers, Government Surveyor, Cape Town, South Africa; and

John Watson, Norman Villa, Glossop Road, Sheffield,

were balloted for and duly elected Fellows of the Society.

The following Candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

John Harold Herbert, Civil Engineer, Mapperley Road, Nottingham (proposed by A. A. Common).

Eighty presents were announced as having been received since the last meeting, including, amongst others:—

J. C. Clancey, Aid to land-surveying (two editions); Calculating Tables; Examination questions in surveying, presented by

the Author; Galileo, Opere, Edizione nationale, vol. vi., presented by the Italian Government; *Nautical Almanac* for 1900, and ditto, Part I., presented by the Lords Commissioners of the Admiralty; S. Newcomb, The Mass of *Jupiter* and the Orbit of *Polyhymnia* (American *Nautical Almanac* papers, vol. v. part 5), presented by the American *Nautical Almanac* Office; *Washington Observations*, 1890, Appendix I.; Catalogue of 16,748 southern stars, deduced from observations made by Lieut. Gilliss, presented by the Observatory; Positive and negative reproductions of a photograph of Nebula M 8 *Sagittarii*, presented by K. D. Naegamvala; Photograph of the partial phase of the solar eclipse 1896 August 9, taken at Vadsö, presented by Joseph Lunt.

ALTERATION OF BYE-LAWS.

At the Special General Meeting held after the Ordinary Meeting, the following alteration of Bye-laws was unanimously carried:—

That Bye-laws 69 to 74 be altered by the addition of the word "gold" before the word "medal."

The reasons for, and effect of, this alteration are detailed in *Monthly Notices*, vol. lvii. pp. 35–39.

The Bye-laws in question now stand as follows:—

SECTION XVI.—*Of Awarding the Gold Medal.*

69. No more than one gold medal shall be awarded or given during the course of any one year.

70. At the regular meeting of the Council, in the month of November in every year, the Council shall (prior to any other business) take into their consideration the propriety and expediency of awarding such gold medal; and the name of every person who may be proposed and seconded as deserving the same shall be entered in the Minute-book.

71. Whenever more than one person shall have been proposed, the Council shall, at their regular meeting in December, select one of the persons so proposed to be submitted to the Council in January following on the question whether he shall or shall not receive the gold medal.

72. At the regular meeting of the Council in January following, the Council shall decide whether the nominee then before the Council shall or shall not receive the gold medal.

73. No gold medal shall be awarded in any other way than in the manner and at the time above prescribed; nor unless three-fourths at least of the Members of the Council present shall agree thereto.

74. Notwithstanding the preceding Bye-laws, in cases where two or more persons have been jointly concerned in the produc-

tion of any scientific treatise, or the carrying out of any research, work, or discovery, or have been the simultaneous but independent authors of any such treatise, work, research, or discovery, the Council may, under these circumstances, receive the nomination of such two or more persons as joint recipients of the gold medal, and proceed thereupon in like manner as above mentioned with respect to the names of separate persons ; and should the gold medal be ultimately awarded to such joint authors, workers, or discoverers, an impression of the gold medal shall be provided for, and given to each of such joint recipients.

The Total Solar Eclipse of 1896, *August* 9. *Reports of Observers communicated to the Joint Permanent Eclipse Committee of the Royal Society and Royal Astronomical Society.*

The following Reports have been received, and communicated by the Committee to the Royal Astronomical Society for publication. Mr. Stone's report on observations made in Novaya Zemlya will be published in the *Memoirs*.

Report on the Expedition to Japan to observe the Total Solar Eclipse of 1896, *August* 9.

By W. H. M. Christie, M.A., F.R.S., Capt. E. H. Hills, R.E., and H. H. Turner, M.A., B.Sc.

1. Thick clouds obscured the Sun at the time of Total Eclipse, and no photographs were taken or observations made. But the following notes on the Expedition may be interesting to other observers:—

2. This Expedition was organised by the Joint Permanent Eclipse Committee of the Royal Society and Royal Astronomical Society, funds being provided from a grant made by the Government Grant Committee. The observers are indebted to the Canadian Pacific Railway Company and the Allan Line for a considerable reduction of fares, and for the free carriage of their instruments from Liverpool to Yokohama and back to Montreal. By the kindness of the Lords Commissioners of the Admiralty the Expedition was assisted in Japan by several of Her Majesty's ships of war, as mentioned in detail in the next paragraph.

3. *Personal.* — The following persons took part in the Expedition:—

W. H. M. Christie, M.A., F.R.S., Astronomer Royal.
H. H. Turner, M.A., B.Sc., Sec. R.A.S., Savilian Professor of Astronomy at Oxford.
Captain E. H. Hills, R.E., Instructor in Chemistry and Photography at the School of Military Engineering, Chatham.

In what follows these are usually designated by the initials C., T., H. respectively.

It was originally proposed that Captain W. de W. Abney, C.B., F.R.S., should also take part in the Expedition, in which case the observers would have divided into two pairs, and occupied two stations. But Captain Abney ultimately found it impossible to go to Japan, and the number of observers was thus reduced to three. The Astronomer Royal could not reach Yokohama before July 27, owing to the necessity for his presence at Greenwich on Visitation Day (June 6), but the greater part of his equipment was taken out and set up by the other observers, who left three weeks earlier. Under these circumstances it was not found practicable to occupy two stations, and no attempt was made to do so. Even with this simplified programme, the whole of the time available for preparation was fully employed, and the observers feel that the restriction was justified by their experience.

4. *Itinerary.*—The observers T. and H. left Liverpool in R.M.S. *Sardinian* (Allan Line) on May 28, arriving at Montreal on June 8.

They reached Vancouver on June 21 ; embarked on R.M.S. *Empress of Japan* the following day, and arrived at Yokohama on July 5. They found H.M.S. *Humber* waiting to convey them to the selected station in the island of Yezo. A few days were spent in some official visits and in discussing the most suitable stations, and the observers embarked on the *Humber* on July 13, arriving at Akkeshi on July 16. In the bay five of H.M. ships were already anchored, under the command of Vice-Admiral Sir Alexander Buller, on the flag-ship *Centurion* ; and the observers have to gratefully record the generous assistance given them by the Admiral and the officers of the fleet. Three ships in all rendered special aid to the Expedition, viz., H.M.S. *Humber*, which conveyed the observers T. and H. to Akkeshi ; H.M.S. *Linnet*, which conveyed the Astronomer Royal thither from Yokohama, and the officers and men of which assisted the observers in their programme for totality ; and H.M.S. *Pique*, which conveyed the whole party back to Yokohama. It may be mentioned that the chief reason for this division of labour was the breaking down of the Japanese railway from Yokohama to Aomori. The Astronomer Royal had proposed to travel by rail to Aomori in order to save time, and the *Humber* was to meet him there, but the above-mentioned break-down necessitated a change of plans, and fortunately the *Linnet* was able to bring him direct from Yokohama, remaining with the observers till after the eclipse. For conveying all the observers back again, the Admiral sent up the *Pique*. It will be seen how much the Expedition owes to the readiness with which Admiral Buller met an unforeseen contingency.

The Astronomer Royal left Liverpool in R.M.S. *Laurentian*, on June 18, arriving at Montreal on June 29. After some delay,

through floods in the Rocky Mountains, he reached Vancouver on July 12, and sailed in R.M.S. *Empress of China* on July 13, arriving at Yokohama on July 26; left Yokohama in H.M.S. *Linnet* on July 29, arriving at Akkeshi on the morning of August 1, and was landed at once.

It may be added that the day after the arrival of the observers T. and H. at Akkeshi was spent in exploring the neighbourhood and selecting a site for the station. An American party, with Professor Schaeberle at the head, and a Japanese party under Professor Shin Hirayama, of Tokio Observatory, were already established. After consultation with them, and inspection of the neighbourhood, the observers had little difficulty in choosing a site, and arranged to land their instruments in a "sampan" (or Japanese flat-bottomed boat, which could be run up on the beach) the next morning. But owing to dense fog the "sampan" failed to find the *Humber*, and put back to shore after a four hours' search. On the morning of July 19, however, all the apparatus was safely landed, and work was commenced at once.

5. *Selection of a Station.*—There were three sets of stations available:

(*a*) Stations on the north-east coast: Soya, Esashi (Yessassi), Mombets, &c.

(*b*) Stations inland: Shibetcha, &c.

(*c*) Stations on the south coast: Kushiro, Akkeshi, Hamanaka.

(*a*) Were perhaps best as regards weather (see below), but were condemned by the Hydrographer as regards anchorages, which are bad. The ship could not with any certainty have remained with the observers.

(*b*) Meteorological observations showed that Shibetcha was very bad. An opinion was expressed by Dr. Knipping, of Hamburg, and communicated to us by Professor D. P. Todd, that if an observer could get inland from Kushiro to the lee side of the Sulphur Mountains he would have perhaps the best chance of weather. But this would have been a considerable undertaking, requiring much time, and was not attempted. With regard to other inland stations, it may be remarked that the island of Yezo is thickly wooded inland.

(*c*) Kushiro was originally selected as the headquarters of the Expedition, on the advice of Professor Milne, F.R.S., who knew the district. But just as the observers T. and H. were leaving England, information was received from the Japanese Hydrographic Office, recommending the north-east coast (stations *a*) rather than the south coast (stations *c*), on the ground that the prevailing winds, being southerly, blew up fog to the latter stations. At the same time our own Hydrographer condemned the anchorage at Kushiro, and suggested Akkeshi or Hamanaka instead. On arriving at Yokohama the observers had thus to

face the question whether to proceed to Akkeshi or to attempt an entire change of programme by going to some place on the north-east coast, in accordance with the suggestions of the Japanese Hydrographic Office.

The observers have here to acknowledge the valuable help accorded them by the British Minister, Sir Ernest Satow, who was able to obtain for them just the information they required. He first put into their hands a pamphlet by Professor Nakamura, Director of the Central Meteorological Observatory in Tokio, wherein meteorological observations at different stations in Yezo were tabulated and discussed ; and secondly, found for them a Japanese who knew the island of Yezo well, was an excellent navigator, and could speak very good English. From both sources of information it appeared that Akkeshi was equal, if not superior, to the north coast. Further, there were already three strong parties on the north-east coast (Professor D. P. Todd, M. Deslandres, Professor Terao), so that observations there would be well provided for. The decision was accordingly taken to proceed to Akkeshi (spelt Akishi on the Admiralty Chart).

A brief extract from Professor Nakamura's pamphlet is here appended :—

" The undermentioned table shows a cloud condition near the Sun's disc, which has been found from the average of the tri-daily observations (2 P.M., 2.30 P.M., and 3 P.M.) extending over three years, 1893–1895, from July 25 to August 25. Cloud conditions are denoted by five symbols :—

0	means	entirely free from clouds.
1	,,	generally scattered.
2	,,	clouds massed about the Sun.
3	,,	the Sun in fog or haze.
4	,,	the Sun invisible in thick clouds.

The number of observations, in percentage, distributed under each symbol, is given in the following table :—

	0			1			2			3			4		
Name	2.0	2.30	3.0	2.0	2.30	3.0	2.0	2.30	3.0	2.0	2.30	3.0	2.0	2.30	3.0
Soya	27	25	25	21	24	21	10	11	8	5	5	5	36	34	40
Esashi	56	56	56	12	12	12	6	6	6	4	4	4	21	21	21
Mompetsu	58	57	55	9	7	8	5	6	5	1	2	2	23	23	26
Abashiri	30	27	26	10	12	13	5	6	6	0	0	0	54	54	54
Nemuro	23	20	20	17	17	16	10	11	13	9	11	11	40	40	39
Akkeshi	40	41	41	31	28	27	2	1	2	8	9	9	18	18	18
Shibetcha	20	17	17	8	12	11	26	26	24	0	0	0	46	48	48

If we regard cloud conditions which are denoted by 0 and 1 to be suitable for observation and the others unsuitable, then the percentage at the observation hour becomes as follows :—

	2.0 P.M.		2.30 P.M.		3.0 P.M.	
	Favourable.	Unfavourable.	Favourable.	Unfavourable.	Favourable.	Unfavourable.
Soya	48	52	49	50	46	53
Mompetsu	69	30	67	33	66	34
Esashi	68	31	68	31	68	31
Abashiri	40	59	39	60	39	60
Nemuro	40	59	37	63	36	63
Akkeshi	71	28	72	28	71	29
Shibetcha	28	72	29	71	28	72

Below is given the record of weather at the time of totality at Akkeshi during the stay of the observers :—

July 19	Cloudy.	July 27	Fine.	Aug. 4	Fine.
20	Cloudy.	28	Cloudy.	5	Fine.
21	Cloudy.	29	Cloudy.	6	Cloudy.
22	Very fine.	30	Fine.	7	Cloudy.
23	Fine.	31	Doubtful.	8	Very fine.
24	Doubtful.	Aug. 1	Cloudy.	9	Cloudy.
25	Cloudy.	2	Cloudy.	10	Very fine.
26	Fine.	3	Raining.	11	Fine.

On arrival in Akkeshi Bay it was not difficult to make a choice of station, aided by the experience of Professor Schaeberle, who had already been ashore eleven days. He had remarked that the sea-fog (which was only too prevalent throughout our stay) lay about the hills and inland when it was clear in the little basin in which the village lay. We accordingly put up at a tea-house, or inn, in the village, and erected our huts in the yard at the back within a ring-fence, which kept out not only inquisitive human beings, but dogs and poultry. The Governor of the place and the Chief of Police at once took us under their protection, and we were never interfered with in any way, but on the contrary received every kindness. We had never any reason to regret our choice on the score of weather. We often had it beautifully fine when there was dense fog at sea. For instance, on the day before the Eclipse, the Sun shone uninterruptedly from a clear blue sky, while Admiral Buller, who had come up in H.M.S. *Alacrity* to see the Eclipse, was kept outside the bay in a dense fog only a few miles away, and did not venture to come in till the Monday morning after the Eclipse. On the actual day of the Eclipse we could not have been better situated in the neighbourhood, for the fog and cloud did lift to us occasionally, though it remained dense elsewhere.

6. *Instrumental Equipment.—The Astronomer Royal.*—The "Thompson" photoheliograph : O.G. 9 inches aperture and

8 ft. 6 in. focal length, with secondary magnifier placed a short distance within the focus, giving an image of the Sun 4 inches in diameter. The telescope was pointed to a 16-inch plane mirror made by Dr. Common, and mounted as a cœlostat by Mr. Hammersley after design by Dr. Common ; the Sun's rays being thus reflected into the telescope. The camera was furnished with eight plate-holders, taking 12 × 10-inch plates.

Professor Turner.—The double camera used in the Fundium Expedition of 1893 by Sergeant Kearney. The tube is of wood, 6 feet long and 14 × 7 inches in section, divided by a partition into two square tubes of 7 × 7-inch section. In one of these is placed the "Abney"* lens of 4 inches aperture and 5 feet 2 inches focal length, giving an image of the Sun 0·57 inch in diameter ; in the other the photoheliograph objective No. 2 (used in transit of *Venus* expeditions), of 4-inch aperture and 5 feet focal length, with a Dallmeyer secondary magnifier of $7\frac{1}{2}$ inches focus placed 5 inches within the focus, and giving an image of the Sun $1\frac{1}{2}$ inch in diameter. The camera furnished with six plate-holders, each taking two plates of 160 × 160 mm., as in use for the astrographic chart ; both plates being exposed by a quarter turn of one shutter. The camera pointed to a 16-inch cœlostat as above.

On the tube was mounted a polariscopic apparatus consisting of an ordinary slit spectroscope (and telescope), with an Iceland spar double image prism substituted for the ordinary prisms. Further explanation of this apparatus is given below.

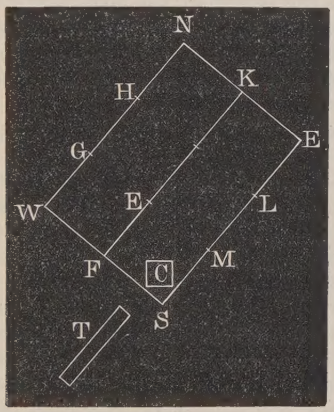
Captain Hills.—Two slit spectroscopes :

	No. 1.	No. 2.
Objective	Cooke achromatic $4\frac{1}{2}$ inch aperture, 5 feet 10 inch focus.	Single quartz lens 5 inch aperture, 4 feet 9 inch focus.
Collimator and Camera lenses	Single quartz lens $2\frac{1}{2}$ inch aperture, 30 inch focus.	Single quartz lens 3 inch aperture, 36 inch focus.
Slit	$1\frac{1}{2}$ inch × 0·004 inch.	2 inch × 0·004 inch.
Prisms	Two flint prisms of 60° $4\frac{1}{2}$ inch base, $2\frac{1}{2}$ inch height.	Four quartz prisms of 60° $3\frac{1}{4}$ inch base, $2\frac{3}{4}$ inch height.
Prisms at min. dev. for	Hγ.	Hϵ.
Position of slit	Tangential to Sun's limb at point of second contact.	Diametral.

* One half of a doublet photographic lens by Dallmeyer belonging to Captain Abney used in the Eclipse Expeditions of 1886, 1887, 1889, 1893, and 1896 : acquired from him by the Royal Astronomical Society in 1893 for permanent use in Eclipse expeditions, in consideration of their replacement by two others.

7. *Huts.*—The huts for the instruments were made of wooden frames covered with Willesden waterproof canvas. These were made before starting, and taken out with the instruments. The canvas was brought away after the eclipse, and the wood left behind. This procedure is recommended for future expeditions.

The huts for observers C. and T. were arranged on the plan suggested in *Monthly Notices*, vol. lvi. p. 410. The corners were placed N., E., S., and W., and the cœlostats (C.) mounted on foundations of gravel and Portland cement in the south corner. Two sliding doors, MS and FS, met at the south corner, admitting the Sun to the cœlostat in the morning and afternoon respectively. The telescopes (T.) were placed in the azimuth indicated in the diagram. The distances SF, FW, WG, GH,



HN, &c., were all about 4 feet, slightly less than the width of canvas. There was a ridge roof, with ridge along the line KF ; and T.'s hut was completely partitioned along FK to form the dark-room in the portion WNKF, a lining of brown paper being used to form the dark-room. The paper used was such as is put under carpets, with a pattern of depressions in it to make it soft to the feet ; but these depressions or bulges were found to be easily pierced, and thus let in light ; ordinary brown paper would have been better.

The arrangement was found to work well on the whole.

H. adopted a different plan. His heliostat and object-glasses were placed outside the main hut on a concrete foundation, and were protected by a small structure of the same wood and canvas, which could readily be thrown off. Inside the main hut was placed a firm table on which the spectroscopes were mounted. The tubes to connect the object-glasses with the spectroscopes were made of black paper on wooden laths on the morning of the Eclipse. This arrangement also was found to work well.

8. *Mounting and adjustment of Cœlostats and Heliostat.*—The places for the huts having been marked out on the

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ground, the cement bases were made at once, so that they might have time to set while the huts were being put up. Rectangular wooden frames were made, sunk an inch or two into the ground (which was tolerably hard), and filled with shingle and Portland cement. A layer of sand and cement was placed on the top to give a good surface. Before the cement was quite hard the heavy metal bases for the cœlostats were placed in position roughly, so that they might bed into the cement.

The adjustment of the axes of the cœlostats was effected very quickly by means of the attached declination theodolites. The level attached to the telescope makes it possible to adjust in altitude without any astronomical observation, for the latitude of the place can be taken from the chart with sufficient accuracy ; and setting the telescope to the south declination equal to the co-latitude, and in the meridian, the level should indicate horizontality. Index errors of the circle and level are eliminated by reversal of the instrument. There is a slight uncertainty attending the placing of the telescope in the meridian, but this does not seriously affect the adjustment in altitude. If a cross-level were made for the pivots of the telescope this uncertainty could be removed.

To adjust in azimuth we must have an observation of the Sun (or a star) at a distance from the meridian. Observing his declination (in reversed positions of the instrument and taking the mean), the instrument must be moved in azimuth until this observed declination agrees with that given in the *Nautical Almanac*. A very few trials, if the Sun can be seen for half-an-hour, will soon indicate the true azimuth without any calculations, within a minute or two of arc, though if the instrument is moved much the altitude observation should be repeated.

After the initial adjustment of the cœlostats they were not disturbed, but observations were repeated at intervals with the following results. The individual readings were only taken to minutes of arc, but both limbs of the Sun were observed in both positions of the instrument, and the mean of the four set down.

C.'s instrument : Observer T.

Date.	(G.M.T.)	Hour angle of Sun.	Observed Decl.	Tabular Decl.	O−C.
	h	h	° ′	° ′	′
July 20	19·7	+5·0	20 23·0	20 22·7	+0·3
21	9·7	−5·0	20 14·3	20 15·6	−1·3
21	17·5	+3·2	20 10·5	20 11·8	−1·3
22	12·0	−4·3	19 59·8	20 2·3	−2·5
25	17·7	+3·4	19 18·5	19 20·4	−1·9
25	19·0	+4·7	19 18·8	19 19·7	−0·9
25	19·5	+5·2	19 19·8	19 19·4	+0·4
26	10·7	−3·6	19 9·0	19 11·0	−2·0
29	11·5	−2·8	18 25·5	18 28·2	−2·7
29	18·8	+4·5	18 24·0	18 23·8	+0·2
31	11·3	−3·0	17 57·8	17 58·7	−0·9

From this general watch kept on the instrument, it seemed clear that the adjustment remained good within 2′ or 3′, which is sufficient for the purpose. The error was chiefly in altitude, and by an unfortunate oversight, observations with the level for altitude were not recorded, though one or two were made, and the history of this small displacement cannot be traced. But observations made by C. after his arrival gave the following results :—

Date.	(G.M.T.)	Hour angle of Sun.	Observed Decl.	Tabular Decl.	O−C.
	h	h	° ′	° ′	′
Aug. 7	19·3	5·0	16 0·7	15 59·7	+1·0
7	19·3	observation with level			−2·7

T.'s Cœlostat : Observer T.

This instrument was used with the driving-rod and gear at the upper end of the axis, which brings the theodolite to the lower end. The adjustments were made before mounting the mirror in the working position, and could not well be tested afterwards without reversing the mirror. Hence we have no information between July 22 and August 3, when a displacement in altitude was found as in the case of C.'s instrument.

Date.	(G.M.T.)	Hour angle of Sun.	Observed Decl.	Tabular Decl.	O−C.
	h	h	° ′	° ′	′
July 21	18·3	+4·0	20 14·0	20 11·4	+2·6
22	10·3	−4·0	20 4·8	20 3·2	+1·6
Aug. 3	12·0	−4·3	17 17·0	17 11·3	+5·7
3	12·0	observation with level			+8·5
3	17·0	+2·7	17 13·8	17 8·0	+5·8
		Instrument adjusted in altitude.			
3	17·5	observation with level			+1·0
3	18·0	+3·7	17 7·8	17 7·3	+0·5
7	15·0	+0·7	16 8·8	16 2·8	
7	15·0	observation with level			+2·0

H.'s Heliostat : Observer T.

This instrument not having any special means of adjustment provided, the following plan was used to adjust the polar axis.

A wooden cap was made for the end of the polar axis on which one of the cœlostat theodolites could be fitted, and the whole was bound up firmly with string, as the best method of mounting the theodolite on this axis at command. The adjustment was thus conducted as for the cœlostat. The nature of the mounting gave a little extra trouble, but when some few difficulties had been overcome the adjustment was successfully carried out. It is not so important for spectroscopic observations that the adjustment should be perfect, and the labour of re-testing at a later date was omitted.

9. *The Driving Clocks.*—The clocks for the two cœlostats seem unnecessarily heavy in construction, considering the small amount of work to be done, but it does not seem impossible that they may be made to work well. As a matter of fact, T.'s clock gave a good deal of trouble, though the mechanism was not directly to blame. On July 31, after having performed very well day after day, it suddenly stopped, and could not be made to go, even with much extra weight. It was taken to pieces by T. and carefully cleaned, and went well for one day, when it stopped again. A scrutiny revealed the fact that the great weight required to drive the clock had gradually strained the wooden support so that the weight-cord was now rubbing tightly against the side of the opening from the clock-case, producing great friction. The clock was moved about two inches forward and the support strengthened, and all went well for another day, two weights being now sufficient for driving instead of four. Then came another stoppage, and five weights would not drive it. It was again taken to pieces and thoroughly examined by T. and H., when it was found that the upper pivot of the governor would jam unless the plate in which it worked was pressed tightly to one side before screwing it down. When this was done the clock ran easily, and continued to do so until the day of the Eclipse. The primary cause of the series of difficulties was almost certainly the great weight required to drive the clock, which strained the support in the first instance.

The cœlostat clocks, when in good order, kept a fair rate. The governor makes a slight noise twice every second, by which the rough adjustment of rate can be made. For fine adjustment a mark was placed on one of the wheels, which revolves four times a minute, and transits of this mark were observed at intervals ; or the image of the Sun was received on the ground glass of the telescope by reflection from the cœlostat, and its steadiness observed. The two methods were found to agree satisfactorily, showing that the radius of the driving sector is of the right length or rather that the teeth are properly cut. The clock stops when winding, which is an inconvenience. For instance, on the actual day of eclipse the Sun appeared about half an hour before totality, and the image was adjusted on the ground-glass. It was important not to lose it again, and so the clock was kept running ; but having to be wound up some minutes before totality, the image was displaced by the stoppage of the clock. The time lost was, however, only a few seconds, and the displacement not serious. It might even have been corrected if necessary in the following way : before winding, the mark on the wheel was transiting at 6^s, 21^s, 36^s, and 51^s in each minute; after winding, at 13^s, 28^s, 43^s, and 58^s. The rate of the clock might have been increased (by a gentle twist on the driving rod) until the transits again occurred at 6^s, 21^s, 36^s, and 51^s. Or the ordinary slow motion might have been used if its relation to the clock rate was known ; it could not of course be used in the ordinary way so long as the Sun was not visible.

10. *Focussing of Telescopes.*—One of the uncertainties that harass the eclipse-observer is his ignorance of the amount of fine weather he will have before the eclipse for adjusting. He must have some astronomical observations to set up his equatorial in position, and he has been accustomed to depend upon star photographs for his focussing. But the experience of the present Eclipse, and especially the introduction of the cœlostat into eclipse work, has done something towards making him independent of weather. The adjustments for position have been simplified by the introduction of the attached level, and experience has shown that the instrumental azimuth will remain steady, so that one glimpse of the Sun or a known star is sufficient to set up the instrument, and it has been found that the focus can be very conveniently and accurately obtained with the help of the plane mirror and without any astronomical observations at all. It is not the first time that methods of focussing by terrestrial means have been proposed or tried. Dr. Schuster proposed in 1886 (*Phil. Trans.* vol. 180 (1889), Ap. 302) to focus on a terrestrial object at a moderate distance. With a properly constructed tube it should be possible to do the focussing before leaving England, though the experience of this Expedition shows that it is advisable to confirm it on arrival at the station. There should in future be no difficulty about doing this in the manner described below, and the writers of this report feel that the introduction of the cœlostat into eclipse photography has simplified the problem very sensibly.

The method was briefly as follows: Pointing the telescope normally to the mirror, the image of a bright point in the focal plane will be formed in the focal plane. If the bright point be outside the focal plane its image will be within, and *vice versâ*. Now prepare a special spare back for one of the plate-holders with holes in it, through which the light of a candle may be admitted. To form a definite image, wire gauze should be placed in the holes, or some other device used, and the holes should be confined to one-half of the back. Cutting a plate in two, place one-half in the plate-holder where the back is not perforated; then when light is admitted through the holes, a reflected image will be photographed on the half-plate.

Wire gauze placed in the plane of the film was used to form the image by C. Two different devices were used by T. In the first a slip of glass, 3 inches long by $\frac{1}{2}$ inch wide, coated with black varnish, and then ruled with fine lines, was held firmly by the back of the plate-holder in a plane *inclined* to that of the film, one end being in the plane of the film and the other about an inch from it. Only one point of any of the ruled lines could thus give a good image at the same exposure, but from the position of this point it could be inferred how much further the object-glass must be moved. In the second method the half of an old plate was coated with black varnish, ruled with lines, and placed in the unoccupied half of the plate-holder. It was

thus made quite certain that the origin of light was in the plane of the film. Other devices will probably occur to others.

If the image is not in focus, the object-glass (or the secondary magnifier) must be moved until it is, for in the tubes used in this Expedition the plate-holder came against a fixed bearing. It was found possible to focus with considerable accuracy, a displacement of 0·01 inch in the secondary magnifier making a sensible difference. To move the magnifier through such small distances, C. used thicknesses of paper cut in the shape of collars, and placed between two flanges arranged for the purpose, one on the tube holding the magnifier, and the other on the tube into which it slid. T. unscrewed the object-glass through a fraction of a turn, having no flanges available.

The following are the details of the observations for focus :—

(*a*) *The Thompson Photoheliograph.* Observer C.—Focussing of Thompson photoheliograph, 1896 August 4. Tried focus as found at Greenwich, with ring on concave magnifier marked right focus, using method by reflection from mirror, the telescope being in east direction nearly horizontal. Found focus required alteration ; put in two rings of zinc and two of paper, and took another photograph. Lens required to be moved out farther. Four photographs for focus were then taken on this and following days :—

(1) Four thicknesses of note paper added to the two of zinc and paper. August 4.

(2) Two thicknesses of cartridge paper added. August 7. Focus very nearly right.

(3) Two thicknesses of brown paper added. August 7. Focus much worse than No. 1.

(4) *One* thickness of cartridge paper added to No. 1. *i.e.* the two thicknesses of brown paper and one thickness of cartridge paper removed. This gave the best focus, better than No. 1, and was adopted August 8.

N.B.—The object photographed was the hole with net over it, just above the centre of the field, and the reflected images were in a line a little below the centre. A cross is scratched in right-hand lower corner, as placed in telescope.—W. H. M. C.

(*b*) *The Abney Lens.* Observer T.—D = distance of rim of cell of object-glass from end of wooden tube in inches. The film was 6 feet $\frac{1}{16}$ inch from the same end of tube by measurements such as can be made with a 2-foot rule.

On August 3 three exposures were made with the inclined slit : D = 10·60 inches, 10·80 inches, 11·00 inches.

Calling the end of the slit which is in the plane of the film

E, and a point distant 1 inch from E in the plane of the slit E+1, then when

D = 10·60, E was in fair focus.

D = 10·80, E+1·3 was in fair focus.

D = 11·00, E+2·0 was in fair focus.

Now the inclination of the slit was 1 in 3; hence E+1·3 was 0·43 inch distant from the film, and the plate or object-glass should be moved through half this amount to get good focus. Hence the values of D indicated by the three exposures are—

(1) . . . . . 10·60 − 0·0/6 = 10·60

(2) . . . . . 10·80 − 1·3/6 = 10·58

(3) . . . . . 11·00 − 2·0/6 = 10·67

D was set as accurately as possible at 10·65, by pushing or pulling the sliding tube, which fitted rather tightly. But as the first exposure above seemed to indicate a smaller value for D than this, and as star-trails taken on August 4 at this reading, though very good, seemed possibly capable of improvement, another plate was exposed on August 8 in three positions of the O.G., determined by unscrewing it slightly, the first having it hard up, the second unscrewed an eighth turn, the third a quarter turn. For this plate the inclined slit was removed and a half-plate blackened, and then scratched, placed in the carrier as described above. The individual images were thus: the first and second were equally good, and the third distinctly worse. Accordingly the O.G. was unscrewed $\frac{1}{16}$ turn as the adopted position for the Eclipse.

(25 turns = 1 inch.)

(*c*) *The Dallmeyer Lens.* Observer T.—(First plate). The object-glass was unscrewed through fractions of a turn (12 turns to the inch, 1 turn = 0·08 inch), from a position in which its rim was 1″·90 from the end of the tube, the rim of the magnifier being 14″·50 from the *other* end.

1st exposure O.G. hard up.

2nd exposure O.G. unscrewed $\frac{1}{2}$ turn = 0·04 inch.

3rd exposure O.G. unscrewed 1 turn = 0·08 inch.

By some accident the first was not on the plate. The second and third were both fair, the second rather better. Another plate was exposed for confirmation, and all three images were on the plate, second and third being about equal, first distinctly worse. Thus a position of the O.G. about 0·06 inch or 0·05 inch further from the enlarging lens was indicated. There is no slide tube for the O.G., and as it was not considered advisable to unscrew it so much as this, the magnifier was moved $\frac{1}{16}$ inch nearer the plate by its slide tube, and clamped up tight. In this position satisfactory star-trails were obtained in the evening of August 4. But for final confirmation another plate was exposed by candle-

light on August 8 with O.G. home, unscrewed $\frac{1}{8}$ turn and $\frac{1}{4}$ turn. The third was distinctly worse than the other two, and the first slightly better than the second. Hence the O.G. was screwed home for the Eclipse.

It may be noticed that no mention has been made of the inclined slit in connection with the Dallmeyer lens, though it was used for the earlier photographs. The fact is its graduation is not so effective when there is an enlarging lens. The distance that must be accurately adjusted is that between O.G. and magnifier, and a slight change in the distance of the plate from the combination does not matter very much. The displacement of the plate is in fact m^2 times the corresponding displacement of the magnifier, where m is the magnifying power.

The above measurements, being made with a foot-rule, do not claim any great accuracy, but they should be sufficient to give anyone who uses the tube in the future an approximate focus on which he may refine.

Several plates not mentioned above were more or less failures, such as are common in trying a new method, however simple.

Besides the star-trails on August 4, with both instruments, two plates with six exposures each were taken of the Moon on July 30 : O.G. rim 1·90 inch from end of tube, as above, and the rim of the magnifier 14 and $x/16$ inches from the other end, where x had the values :

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
1st plate	8	10	12	8	6	2
2nd plate	8	7	6	9	10	(cloudy)

In the first plate (*d*) and (*e*) were fair.

In the second plate (*b*) was the best, agreeing with the reading subsequently found by reflection from the mirror. Such confirmation of the focus by astronomical exposures is eminently desirable when possible, but it is comforting to be able to dispense with it ; for in many climates there is a heavy dew at night, which is anything but good for the instruments, as was the case on the present occasion at Akkeshi.

11. *Programme of Observations.*—C.'s eight slides were filled, and intended to be exposed as below :—

No. of Slide.	Exposure. s	Plate.	Developer to be Used.
1	1	Ilford ordinary	Pyro-soda.
2	5	Ilford rapid	Pyro-soda.
3	5	Ilford ordinary	Pyro-soda.
4	10	Rocket (backed)	Amidol.
5	2	Ilford ordinary	Amidol.
6	20	Rocket (backed)	Eikonogen.
7	10	Ilford ordinary	Pyro-soda.
8	1	Ilford ordinary	Eikonogen.

T.'s six slides were filled as below, the same plates being used for the Dallmeyer and Abney lenses in each case, and standard squares being impressed on plates 1, 2, and 5 :—

No. of Slide.	Exposure. s	Plate.	Developer to be Used.
1	1	Ilford ordinary	Pyro-soda.
2	2	Ilford rapid	Eikonogen.
3	20	Rocket (backed)	Eikonogen.
4	5	Mawson and Swan (backed)	Amidol.
5	2	Ilford ordinary	Pyro-soda.
6	1	Photo mechanical	Amidol.

H.'s four slides were filled as below :—

Spectroscope.	No. of Slide.	Exposure.	Plate.
No. 1	1	Twelve of 1^s	Lumiere green sensitive orthochromatic.
No. 1	2	Two of 80^s and 30^s	
No. 1	3	Twelve of 1^s	
No. 2	4	Two of 80^s and 30^s	Imperial flash-light.

Slides 1 and 3 were to be exposed during the 20 seconds, commencing about the 15th preceding totality, and the corresponding period at the end of totality.

For slides 1 and 2 the slit was tangential at the point of second contact ; for slide 3 the slit was tangential at the point of third contact ; for slide 4 the slit was diametral.

12. *The Polariscopic Observations.* (This paragraph by T.) In the spring of 1894 I made some measures on the photographs of the corona in 1893 with Captain Abney's revolving-sector photometer. A sufficient number of measures was made to determine the general diminution of coronal light with distance from the Sun's limb, but the time at disposal did not allow me to complete the investigation by taking some particular streamer and exploring its detail. In hopes of doing this in the near future, I did not publish the results as far as they went, though a brief account of them was given to the British Association at the Oxford meeting of the same year (B.A. Report, 1894, p. 568).

Meanwhile the simplicity and beauty of this photometric method had so much impressed me that the extension of it to other cases naturally presented itself, and, in particular, it occurred to me that if a photograph of the corona could be taken with polarised light, and the density of deposit measured at different points, we should obtain a very much more complete knowledge of the extent to which the coronal light is polarised than we have at present. In 1878 some small photographs of the corona were obtained, the light being polarised in planes at right angles (*Washington Observations*, 1876, Appendix III., plate 29), but

no attempt at measuring them was made, and the method of obtaining them was rather rough.

In considering the best form of apparatus, I first thought of placing an achromatised double-image prism of Iceland spar in front of the object-glass of a small telescope. This would give two images of the corona polarised in perpendicular planes, but each impressed on the sky belonging to the other as background. If a large separation of the images could be obtained, the interference of the sky belonging to the other image might be nearly uniform, and not very great; but this would mean a great expense, at the present price of Iceland spar. On thinking over the matter further, however, a different method suggested itself. The above arrangement is analogous to the spectroscope with objective prism, which gives a series of coloured images, each impressed on the sky of the others. With the double-image prism there are only two instead of a number, but this difficulty of confusion is common to both instruments. But the colours of the spectrum can be separated from each other, and this confusion avoided, by the use of a different instrument—namely, the spectroscope of ordinary form, telescope, collimator with slit, prism, camera.

If colour is regarded as continuous, only an infinitely small slit will give a pure spectrum. But if any finite number of colours is assigned, there will be a definite width of slit for which each colour is completely separated from the rest. With half the number of colours, the slit may be just twice as wide; if we again halve the adopted number of colours, we again double the width of slit, and so on. If we have only two colours, the slit may be a considerable width, and yet the images in the two colours be completely separated. Now the Iceland-spar prism is practically a direct-vision prism giving only two colours—the two kinds of polarised light. We have then only to replace the prism of a direct-vision photographic spectroscope by our achromatised Iceland-spar prism, and we have an apparatus giving us two images of the corona (or part of it) in polarised lights without interference.

Such an instrument was arranged for the recent Eclipse, but as no results were obtained, it seems unprofitable to describe it in detail here. This brief sketch of the method will probably be sufficient to guide anyone who wishes to try it, and to whom it had not already occurred. Two remarks may be added:

(1) The focus is not quite the same for the two images. It seems advisable to receive them on separate plates, the focus of each being adjusted independently, though some other way of getting over the difficulty may suggest itself.

(2) The angular separation of the images depends on the focal lengths of the lenses employed as well as on the prism. It is easy either to work it out theoretically, or to observe it experimentally, and then adjust the slit so that there is no overlap. The slit of an ordinary spectroscope may be employed if it will open wide enough, but a rougher slit of metal (or even paper if it be remembered not to use it on the uneclipsed Sun) may be used.

13. *Assistance.*—C. and T. each required four assistants: viz., one to expose at the object-glass end, one to record time of exposures, and two to hand and receive the plate-holders in order. The officers of the *Linnet* who assisted were :—

Recorder for C., Lieut. S. M. Agnew
Exposer for C., Lieut. H. A. Gillett.
Recorder for T., Mr. H. O. Jones.
Exposer for T., Mr. R. B. Garde.

At 15^s before totality (estimated by the previously calculated length of the diminishing crescent) T. was to give the signal "Stand by," in order that H. might make spectroscopic exposures near second contact. At totality he was to call sharply, and two signalmen, A and B, with a stop watch and chronometer, were then to count time as follows: A to record the time of the signal by the chronometer, B to start the stop watch and count from it, "One, two, three, &c." up to sixty seconds, in a loud voice; the next minute was to be similarly counted by A, the change of voice reminding the observers of the change of minute, and the 35^s or 36^s of the third minute by B again, A recording the signal given at the end of totality. The exposer exposed at a signal given to him by the observer when the plate was in position, the beginning and end of exposure being noted by the recorder. In the case of the longer exposures the recorder reminded the exposer quietly at what count he was to close up.

H. only required two assistants: an officer to make the exposures with spectroscope 2 (Lieut. Stevenson, R.N.), and a sailor to hand slides.

All these operations were rehearsed several days before the eclipse, viz. on—

August 5, four times, without plates.
August 6, five times, „ „
August 7, three times, „ „
August 8, once with all slides filled, once again without plates.

On August 7 and 8 all the rehearsals passed off without any hitch.

The Day of the Eclipse.—Quite cloudy and foggy in the morning. All was ready and in good order. The Sun appeared at noon through cloud, and was visible at intervals. The partial phases of the Eclipse were seen until about twenty minutes before totality, when the clouds closed in completely. The counting of seconds was begun some ten seconds after totality had commenced, as judged by the darkness; the commencement was not very well marked, but the observers all agreed afterwards that when the signal was given the darkness had been constant for about ten seconds. The end of totality was unmistakable, being sharply marked, when the counter was calling $2^m\ 22^s$.

Report on the Expedition to Vadsö, in Finmark, to observe the Total Solar Eclipse of 1896 *August* 9. By A. A. Common, LL.D., F.R.S.

In accordance with the arrangements made by the Committee I left England on July 25 in the S.S. *Norse King*, arriving at Vadsö on August 2 with my party.

During totality the sky was cloudy, and no observations were made or photographs taken.

The arrangements were made as follows :—

Observers.—My party consisted of Mr. A. Hinks, of the Observatory, Cambridge ; my son, T. A. Common, Undergraduate, Trinity College, Cambridge ; Major MacMahon, F.R.S. ; Mr. W. H. Wesley, Assistant Secretary R.A.S. ; and my workman, Alfred Wooldridge.

At the request of the Secretary of State for India Mr. K. D. Naegamvala, of the Poona Observatory, India, was attached to the party.

Mr. Naegamvala's object in going to Norway was to gain experience in view of the next Total Solar Eclipse, the central line of totality of which passes near Poona.

Mlle. Klumpké, of the Paris Observatory, also accompanied us.

Instruments.—Two cœlostats with 16-inch plane mirrors were taken, one being the property of the Royal Astronomical Society, the other my own. With these it was proposed to use—

1. The twin telescope as used in Para Caru, Brazil, in 1893. This telescope has an achromatic lens of 3·85 inches aperture and 62 inches focal length, giving an image of the Sun about $\frac{5}{8}$-inch diameter, and also a $4\frac{1}{2}$-inch lens, which in combination with a negative lens of about 8-inch negative focus gives an image about $1\frac{5}{8}$-inch diameter. This combination is arranged to give an image in the same plane as the other. Plate-holders carrying two $6\frac{1}{4} \times 6\frac{1}{4}$ plates fit to the end of the twin tube, and are held in place by a clamp during exposure.

Instead of the ordinary slide, which has to be withdrawn to uncover the plate, a flap is used which opens out into the tube, and may, if needed, be used for making the exposures. The single lens has been used in previous eclipses, and is designated the Abney lens. It is one of a pair forming a photographic lens, formerly belonging to Captain Abney, C.B., now the property of the Royal Astronomical Society. An exactly similar twin telescope was made for and used in the 1893 eclipse at Fundium, West Africa.

2. A 6-inch doublet lens by Grubb, the property of, and kindly lent for this occasion by, Mr. F. McClean, F.R.S. This had a similar negative combination to that of the twin telescope arrangement, but the image in this case was $3\frac{1}{2}$-inch diameter.

This instrument was mounted on a firm wooden framework, and provided with two plate-holders, each carrying three 12 × 10 inch plates. A long frame was attached to the framework, which carried the plate-holders in such a way that the plates could be successively brought to the place required by merely sliding them along the frame, the exposure being made by a hinged shutter near the concave lens.

3. A slit spectroscope with $3\frac{1}{2}$-inch lenses and two 60° light flint prisms.

4. A grating spectroscope, with which it was intended to get the photograph of the spectrum of the corona without a slit.

With instrument No. 1 it was proposed to make exposures of

No. of Slide.	Exposure in Seconds.	Plates.
1	3	Dry collodion plates
2	9	,,
3	27	,,
4	12	Cadett ordinary plates
5	6	,,
6	3	,,

This arrangement uses 60^s of the 104 available at this station. Mr. Hinks had charge of this instrument, Major MacMahon acting as recorder, and two sailors of H.M.S. *Active* handing and receiving the plate-holders. Mr. Hinks found by experience that an allowance of 7^s for changing one slide for another was sufficient.

With instrument No. 2 it was intended to make exposures of

No. of Plate.	Exposure in Seconds.	Plate.
1	3	Dry collodion
2	6	,,
3	27	,,
4	20	Cadett ordinary
5	10	,,
6	5	,,

I took charge of this instrument, Mr. J. J. Atkinson acting as my recorder.

The slit spectroscope had a double dark slide carrying two whole plates, one being dry collodion, the other Cadett ordinary, two exposures of 35^s being given. A. Wooldridge had charge of this instrument.

With the grating spectroscope, which was provided with a 12 × 10 Cadett ordinary plate, it was prepared to take photographs with the same exposure as with the large lens, these instruments being fed by the same cœlostat. T. A. Common had charge of this instrument.

The R.A.S. cœlostat was used with the twin telescope and the slit spectroscope.

The instruments had been erected at Ealing, and carefully focussed and marked, the focussing having been done on the Sun through deep blue glass in the case of the instruments Nos. 1 and 2.

As our arrival on August 2 was at midnight nothing could be done till the next morning, when, in the company of Sir R. S. Ball, I visited the Governor (Prebensen) of Finmark. In fixing upon Vadsö as the site I was largely influenced by a very complete report on the weather and other conditions affecting the various possible positions from Vardö in the north to the Tana river in the west, and the various positions on the south side of the Varanger Fiord kindly obtained for me by Mr. Pirrie, the managing director of the Norse King Steamship Company, who made a special trip to Vadsö in the spring.

Governor Prebensen gave the fullest leave to us to choose any site we might think proper. After visiting the observatory of Dr. Copeland, of Edinburgh, who had chosen a site on the hill about $1\frac{1}{4}$ miles north of Vadsö, and the observatory of M. Hasselberg, some few yards to the west of Dr. Copeland, I chose a site somewhat nearer the town and near the toboggan slide.

After the selection of this site we returned to the ship about noon, and found that the instruments had been landed on the quay. As soon as possible all was carted up to the site, and work at once begun.

A wooden house had been made in England in parts that could be easily put together, and this was erected, and the corrugated iron roof put on, in a few hours. All the instruments were safely housed by eight o'clock.

Active work was begun early on the 4th. An iron wire attached to posts driven into the ground served to define the ground we wished to reserve, and was quite sufficient in the case of the natives ; but the small cows that grazed on the moor had not the same respect for our landmark. The instruments were erected on stone foundations, the R.A.S. cœlostat being in the corner of the wooden house, the other in a small shelter made for it.

Open frames of wood were made where the telescopes had to be placed for receiving the light from the cœlostats, and filled with some $1\frac{1}{2}$ tons of large stones. These frames made a most substantial foundation for the instruments, complete steadiness being obtained by piling heavy stones on them when fixed in position. A canvas awning was fixed on poles over each instrument, and was found sufficient protection, but, as stated further on, it would be much better to have the whole under one roof.

Owing to the absence of the Sun and the practical impossibility of seeing a star, considerable difficulty was found in getting the cœlostats into position in azimuth. A rough test we

obtained of the Sun's image at an interval of some hours showed a slight shift upwards in both cases. With constant sunshine there would have been no difficulty in getting the adjustments sufficiently correct in a short time. Where this cannot be depended upon it is necessary to allow much more time than we had at our disposal.

In the preparatory work I had the help of several fellow passengers. H.M.S. *Active*, of the Training Squadron, was in the harbour of Vadsö, and Commodore Atkinson very kindly placed a boat's crew and boat at my disposal, and also gave me the services of Yeoman Signaller Simmons and signalman three days before the eclipse, and four sailors on the day of eclipse. The boat was a very great help, as we left the ship at an early hour each morning. Rehearsals were made on the Saturday, and all was ready by Saturday night.

A drenching rain in the early morning of the day of the eclipse got us into the proper frame of mind to meet the disappointment we experienced some hours later.

A short time before totality was due blue sky was seen in the north, and the clouds began to roll away from the Sun at such a rate that it seemed as if it would clear, but by the time these clouds had got away others had come up from the west and covered the Sun.

Mlle. Klumpké had a telescope erected in the north part of our enclosure, and purposed taking time observations of the contacts, assisted by M. Fano.

Sir R. S. Ball had an equatorial telescope in the south side, and undertook to give the signal to stand by about three minutes before totality, and the time of second contact. On hearing this J. S. Simmons began counting seconds 104 to 0 by the help of a dead-beat seconds watch. I had arranged that all should be done as if the sky was clear, and at the word the exposures were all duly made without the least hitch. The darkness did not roll away when our counter gave the last second, and one had a strange feeling that something had gone wrong, for the darkness and the silence were very oppressive. In a few seconds the light returned and the total eclipse was over for us.

On comparing notes Sir R. Ball came to the conclusion that he had given the signal too soon, the faintness of the image as seen through the clouds quite accounting for this.

On developing the plates on my arrival home we found on the first plate exposed to the Abney lens a thin crescent, evidently the last remaining part of the Sun as seen through thick clouds. A copy of this photograph is sent herewith.

Nearly the whole population of Vadsö—some 1,400—turned out to see the eclipse, a great many coming round us. Major Barlow, R.A., kindly undertook, with the help of a staff of special constables, to keep order if needed, but all behaved in the most charming way.

Mr. Wesley was prepared to make sketches of the corona,

but under the circumstances nothing was done. He and others made observations of the appearance of the sky, &c., but nothing of importance was noted.

With the exception of the want of sunshine for adjustment all the arrangements made answered well. The time at our disposal for setting up the instruments was hardly enough, partly owing to the above reason, but the days were very long, and under ordinary circumstances the time would not have been enough.

The instruments all went well. From want of experience in eclipse work I cannot speak with authority on the matter, but I think that there can be no doubt that the cœlostat is the proper instrument for eclipse work. I can easily see the very great additional work we should have had to do if we had been using equatorial mountings. A great improvement might be made by housing the whole of the apparatus under one roof, suitable openings being left for the requisite sky to be seen from the cœlostat.

The arrangement I used with the large lens for sliding the plate-holder along in a frame is very much better than the plan in use with the twin telescope of putting in and taking out each plate-holder separately, a plan necessary in the case of the equatorial mounting for which the twin telescopes were made ; but if these are to be used again a sliding arrangement should be used, and I am not sure that it would not be better to still further reduce the hand movement necessary now to expose successive plates by having a mechanical arrangement to do all when once started.

Another point. This eclipse shows that all available stations should be occupied if possible ; in this case it would have been well if Bodö had been occupied. On going up the coast I observed the sky in the early morning, and found it very fine indeed, and had it been possible I should have left part of my party at Bodö, Mr. Hinks, in the right scientific spirit, expressing his willingness to forego the pleasures of the longer voyage and to stay there if required. The difficulty of dealing with the instruments separately, however, prevented this being done ; nor could I induce any of the many observers on board to stay behind.

Report to the Joint Eclipse Committee on the Expedition to Kiö Island. By Professor J. Norman Lockyer, C.B., F.R.S.

The party consisted of Mr. Fowler, Dr. W. J. S. Lockyer, and myself. As it was impossible for me to leave London before July 22, Mr. Fowler and Dr. W. J. S. Lockyer went on in advance and joined H.M.S. *Volage* at Hammerfest on July 22,

and left the following day for the Varanger Fjord. On July 24 they were landed at Bras Havn, and spent two days in exploring the neighbourhood for a suitable site.

The erection of the instruments was commenced on July 27, and on my arrival at Kiö Island a week later most of them were in adjustment and ready for the eclipse.

The main objects of the expedition were to photograph the spectra of the solar surroundings with prismatic cameras of 6 and 9 inches aperture, and with an integrating spectroscope having two 3-inch prisms of 60°. In addition, about seventy officers and men of H.M.S. *Volage* were trained to make drawings, read thermometers, note the colours of the landscape, &c., and to assist in the manipulation of the larger instruments.

On the morning of the eclipse the sky was almost entirely overcast, the Sun only being visible for a few seconds during the partial phases, so that all work with the instruments was impossible.

Some useful observations of temperature, landscape colours, &c., were secured by the *Volage* observers; and a report upon these, together with a description of the instruments and observing station, will be communicated to the Committee in due course.

To meet the cost of this section of the expedition, including partial instrumental equipment, a grant of 200*l.* was made by the Eclipse Committee. The accounts are not yet completed, as some of the instruments were slightly damaged and the repairs are not finished. The sum granted, however, has certainly not been exceeded, and it is expected that a few pounds will be left over to return to the Committee.

Thanks are due to the Norwegian Government for permission to land on the south side of the Varanger Fjord, to Captain King-Hall, R.N., and the officers and men of H.M.S. *Volage*, who rendered such splendid assistance during the preparations for the eclipse, and to the Orient Steam Navigation Company for the free carriage of some of the instruments, and for twice altering the course of the s.s. *Garonne* to permit landing and embarking off Kiö.

On arriving home I at once communicated to the Royal Society the efficient aid rendered by the officers and men of the *Volage*, as the captain and a large proportion of the crew were to leave the ship on her reaching port.

On the Variation of Uncanonical Arbitrary Constants; with an application to the Planetary Theory. By A. Y. G. Campbell, B.A., Scholar of Trinity College, Cambridge.

(*Communicated by Sir R. S. Ball.*)

In attacking the planetary or lunar theories by the method of the variation of arbitrary constants, it is necessary to evaluate several functions usually denoted by $[a, e]$, &c., unless a canonical set of arbitrary constants is chosen. In this paper I endeavour to show that the equations giving the variations of these constants can be found more quickly without evaluating the functions $[a, e]$, &c. It is necessary to find one function (denoted in this paper by C with a suffix) corresponding to each arbitrary constant. In the first two sections the theory and its application to the elliptic constants of the planetary theory are briefly explained. In the following section it is shown that if any solution of the new equations is found we can proceed to find the equations, giving the variation of the new arbitrary constants. The next approximation can be obtained in a manner similar to that used in the previous case. It is shown also in the last section that in certain cases there exists a method of solution similar to Jacobi's method of solution of the Hamiltonian canonical equations. Among these cases is included the set of equations giving the variations of the elliptic elements in the planetary theory.

§ 1.

Let $\alpha_1, \alpha_2, \ldots \alpha_{2n}$ be the set of arbitrary constants of any solution of the canonical equations

$$\dot{q}_i = \frac{d\mathrm{H}}{dp_i}, \quad -\dot{p}_i = \frac{d\mathrm{H}}{dq_i} (i = 1, 2, \ldots n) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is required to find the equations giving the rates of variation of $\alpha_1, \alpha_2, \ldots \alpha_{2n}$ when a disturbing function Ω is added to H.

If L is the Lagrangian function corresponding to H,

$$\frac{d\mathrm{L}}{d\dot{q}_i} = p_i, \quad \frac{d\mathrm{L}}{dq_i} = \dot{p}_i (i = 1, 2, \ldots n);$$

and if α_r is one of the arbitrary constants in terms of which L is supposed to be expressed,

$$\begin{aligned} \frac{d\mathrm{L}}{d\alpha_r} &= \sum_{i=1}^{i=n} \left(\frac{d\mathrm{L}}{dq_i} \frac{dq_i}{d\alpha_r} + \frac{d\mathrm{L}}{d\dot{q}_i} \frac{d\dot{q}_i}{d\alpha_r} \right) \\ &= \sum_{i=1}^{i=n} \left(\dot{p}_i \frac{dq_i}{d\alpha_r} + p_i \frac{d\dot{q}_i}{d\alpha} \right) \\ &= \frac{d}{dt} \left(\sum_{i=1}^{i=n} p_i \frac{dq_i}{d\alpha_r} \right) \end{aligned}$$

the terms in the bracket being expressed in terms of the arbitrary constants, &c. In the above $\dot{p}_i$, $\dot{q}_i$ have been written for the values of

$$\frac{dp_i}{dt}, \quad \frac{dq_i}{dt},$$

when p_i, q_i are expressed in terms of $\alpha_1, \alpha_2, \ldots \alpha_{2n}, t$.

It follows that

$$\sum_{i=1}^{i=n}\left(p_i \frac{dq_i}{d\alpha_r}\right) = \int_{t_1}^{t} \frac{d\mathrm{L}}{d\alpha_r} dt + \mathrm{A}_r$$

$\frac{d\mathrm{L}}{d\alpha_r}$ being expressed in terms of $\alpha_1, \alpha_2, \ldots \alpha_{2n}, t$, and integrated with regard to t explicitly; A_r is a function of $\alpha_1, \alpha_2, \ldots \alpha_{2n}$, and does not contain t explicitly; t_1 may be any function of $\alpha_1, \alpha_2, \ldots \alpha_{2n}$, but must not contain t.

Now

$$\frac{d}{d\alpha_r}\int_{t_1}^{t} \mathrm{L}dt = \int_{t_1}^{t} \frac{d\mathrm{L}}{d\alpha_r}\, dt - (\mathrm{L})_{t=t_1} \frac{dt_1}{d\alpha_r};$$

if we put

$$\mathrm{A}_r + (\mathrm{L})_{t=t_1} \frac{dt_1}{d\alpha_r} + \mathrm{B}_r = 0$$

we have

and similarly,

$$\left.\begin{aligned} \sum_{i=1}^{i=n}\left(p_i \frac{dq_i}{d\alpha_r}\right) &= \frac{d}{d\alpha_r}\int_{t_1}^{t} \mathrm{L}dt - \mathrm{B}_r \\ \sum_{i=1}^{i=n}\left(p_i \frac{dq_i}{d\alpha_s}\right) &= \frac{d}{d\alpha_s}\int_{t_1}^{t} \mathrm{L}dt - \mathrm{B}_s \end{aligned}\right\} \quad \ldots\ldots\ldots (2)$$

If we differentiate the first of these expressions with respect to α_s and the second with respect to α_r, on the supposition that all the functions involved are expressed in terms of $\alpha_1, \alpha_2, \ldots \alpha_{2n}, t$, we find that

$$[\alpha_r, \alpha_s] = \frac{d\mathrm{B}_s}{d\alpha_r} - \frac{d\mathrm{B}_r}{d\alpha_s} \quad \ldots\ldots\ldots\ldots (3)$$

where

$$[\alpha_r, \alpha_s] = \sum_{i=1}^{i=n}\left(\frac{dq_i}{d\alpha_r}\frac{dp_i}{d\alpha_s} - \frac{dq_i}{d\alpha_s}\frac{dp_i}{d\alpha_r}\right) \quad \ldots\ldots\ldots (4)$$

Since B_r, B_s are independent of the time explicitly, it is clear that $[\alpha_r, \alpha_s]$ does not contain t explicitly when it is expressed in terms of $\alpha_1, \alpha_2, \ldots \alpha_{2n}$. It follows also that we can assign any value we please to t in equation (2) as far as regards the determination of B_r.

If

$$C_r = B_r + \frac{dA}{da_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where A is any arbitrary function of $a_1, a_2, \ldots a_{2n}$ it is clear that

$$[a_r, a_s] = \frac{dC_s}{da_r} - \frac{dC_r}{da_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If we put $t = t_1$ in equation (2)

$$C = -\left[\sum_{i=1}^{i=n}\left(p_i \frac{dq}{da_r}\right) + L\frac{dt_1}{da_r}\right]_{t=t_1} + \frac{dA}{da_r} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Having thus determined $[a_r, a_s]$, we can now proceed in the usual manner to find the new equations.

Let the original equations be

$$\dot{q}_i = \frac{d(H+\Omega)}{dp_i}, \quad \dot{p}_i = -\frac{d(H+\Omega)}{dq_i} \; (i = 1, 2, \ldots n) \quad . \quad . \quad . \quad (8)$$

Then, since $a_1, a_2, \ldots a_{2n}$ have been so chosen that when $q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n$ are expressed in terms of them and t,

$$\frac{dq_i}{dt} = \frac{dH}{dp_i}, \quad -\frac{dp_i}{dt} = \frac{dH}{dq_i} \; (i = 1, 2, \ldots n)$$

it follows that

$$\sum_{r-1}^{r=2n} \frac{dq_i}{da_r}\dot{a}_r = \frac{d\Omega}{dp_i}, \quad -\sum_{r=1}^{r=2n} \frac{dp_i}{da_r}\dot{a}_r = \frac{d\Omega}{dq_i} \; (i = 1, 2, \ldots n).$$

Multiplying each of these equations by

$$\frac{dp_i}{da_s}, \quad \frac{dq_i}{da}$$

respectively, and adding the complete set, we see that

$$\frac{d\Omega}{da_s} = \sum^{r=2n} [a_r, a_s]\dot{a}_r$$

$$= \sum_{r=1}^{r=2n}\left(\frac{dC_s}{da_r} - \frac{dC_r}{da_s}\right)\dot{a}_r$$

$$\therefore \frac{d\Omega}{da_s} = \dot{C}_s - \sum_{r=1}^{r=2n} \frac{dC_r}{da_s}\dot{a}_r (s = 1, 2, \ldots n) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

are the required equations.

§ 2. *Application to the Planetary Theory.*

The motion of the planets is governed in general principally by the attractive force of the Sun, which would, if acting alone, cause each planet to describe an ellipse. The deviations from the elliptic orbit are small; it is an advantage to deal with variables whose rates of variation are small, for it is then possible to use methods of approximation. In some expositions of the planetary theory the first change of variables is to the constants of the elliptic orbit. The set which are usually denoted by a, e, ϵ, g, α, ϕ, are uncanonical.

Let the origin of coordinates be taken at the Sun's centre, which is supposed to be at rest or moving with uniform velocity in a straight line. If (x, y, z) are the coordinates of the planet referred to three fixed rectangular axes through the origin

$$\mathrm{L} = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{r}$$

when the units are properly chosen.
Then

$$p_1 = \dot{x},\ p_2 = \dot{y},\ p_3 = \dot{z}$$

$$\mathrm{H} = \frac{1}{2}(p_1^2 + p_2^2 + p_3^2) - \frac{1}{r}.$$

Let b denote any one of the constants a, e, ϵ, g, α, ϕ. It is necessary to find

$$\sum_i p_i \frac{dq_i}{db} = \dot{x}\frac{dx}{db} + \dot{y}\frac{dy}{db} + \dot{z}\frac{dz}{dz}.$$

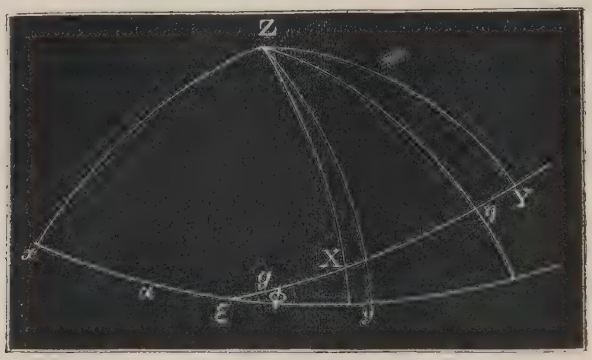
We shall now consider the transformation to new rectangular axes—

$$\left.\begin{aligned} x &= l_1\xi + l_2\eta + l_3\zeta \\ y &= m_1\xi + m_2\eta + m_3\zeta \\ z &= n_1\xi + n_2\eta + n_3\zeta \end{aligned}\right\}$$

where the direction cosines may be functions of the elements, but not of t explicitly.

$$\begin{aligned}
\dot{x}\frac{dx}{db} + \dot{y}\frac{dy}{db} + \dot{z}\frac{dz}{db} &= (l_1\dot{\xi} + l_2\dot{\eta} + l_3\dot{\zeta})\left(l_1\frac{d\xi}{db} + l_2\frac{d\eta}{db} + l_3\frac{d\zeta}{db} + \xi\frac{dl_1}{db} + \eta\frac{dl_2}{db} + \zeta\frac{dl_3}{db}\right) \\
&\quad + (m_1\dot{\xi} + m_2\dot{\eta} + m_3\dot{\zeta})\left(m_1\frac{d\xi}{db} + m_2\frac{d\eta}{db} + m_3\frac{d\zeta}{db} + \xi\frac{dm_1}{db} + \eta\frac{dm_2}{db} + \zeta\frac{dm_3}{db}\right) \\
&\quad + (n_1\dot{\xi} + n_2\dot{\eta} + n_3\dot{\zeta})\left(n_1\frac{d\xi}{db} + n_2\frac{d\eta}{db} + n_3\frac{d\zeta}{db} + \xi\frac{dn_1}{db} + \eta\frac{dn_2}{db} + \zeta\frac{dn_3}{db}\right) \\
&= \dot{\xi}\frac{d\xi}{db} + \dot{\eta}\frac{d\eta}{db} + \dot{\zeta}\frac{d\zeta}{db} \\
&\quad + (\dot{\xi}\eta - \eta\dot{\xi})\left(l_2\frac{dl_1}{db} + m_2\frac{dm_1}{db} + n_2\frac{dn_1}{db}\right) \\
&\quad + (\eta\dot{\zeta} - \zeta\dot{\eta})\left(l_3\frac{dl_2}{db} + m_3\frac{dm_2}{db} + n_3\frac{dn_2}{db}\right) + (\zeta\dot{\xi} - \xi\dot{\zeta})\left(l_1\frac{dl_3}{db} + m_1\frac{dm_3}{db} + n_1\frac{dn_3}{db}\right).
\end{aligned}$$

Let the plane $x\ y$ cut the plane of the ecliptic in the axis ξ, and let the axis η be in the plane of the ecliptic. The angle between x and ξ



is α, and if X denote the axis which is taken along the major axis of the ellipse, the angle between ξ and X is g ; the inclination of the ecliptic to the plane $x\ y$ is ϕ ; a and e have their usual meanings in an ellipse, and ϵ is the epoch.

The solution of the elliptic motion is given by the following equations—

$$\left.\begin{aligned} \mathrm{X} &= a(\cos u - e) \\ \mathrm{Y} &= a\sqrt{1-e^2}\,.\,\sin u \\ a^{-\frac{3}{2}}t + \epsilon &= u - e\sin u \end{aligned}\right\}$$

$$\left.\begin{aligned} \xi &= \mathrm{X}\cos g - \mathrm{Y}\sin g \\ \eta &= \mathrm{X}\sin g + \mathrm{Y}\cos g \end{aligned}\right\}$$

$$\left.\begin{aligned} x &= \xi\cos\alpha - \eta\sin\alpha\cos\phi \\ y &= \xi\sin\alpha + \eta\cos\alpha\cos\phi \\ z &= \eta\sin\phi \end{aligned}\right\}$$

Considering the description of area we see that

$$\xi\dot{\eta} - \eta\dot{\xi} = \mathrm{X}\dot{\mathrm{Y}} - \mathrm{Y}\dot{\mathrm{X}} = \sqrt{a(1-e^2)}$$

Hence

$$\dot{x}\frac{dx}{d\phi} + \dot{y}\frac{dy}{d\phi} + \dot{z}\frac{dz}{d\phi} = 0$$

$$\dot{x}\frac{dx}{d\alpha} + \dot{y}\frac{dy}{d\alpha} + \dot{z}\frac{dz}{d\alpha} = \sqrt{a(1-e^2)}\,.\,\cos\phi$$

$$\dot{x}\frac{dx}{dg} + \dot{y}\frac{dy}{dg} + \dot{z}\frac{dz}{dg} = \dot{\xi}\frac{d\xi}{dg} + \dot{\eta}\frac{d\eta}{dg} = \mathrm{X}\dot{\mathrm{Y}} - \mathrm{Y}\dot{\mathrm{X}}$$
$$= \sqrt{a(1-e^2)}$$

and when $b = a$, e or ϵ

$$\dot{x}\frac{dx}{db} + \dot{y}\frac{dy}{db} + \dot{z}\frac{dz}{db} = \dot{\mathrm{X}}\frac{d\mathrm{X}}{db} + \dot{\mathrm{Y}}\frac{d\mathrm{Y}}{db}.$$

Also

$$L = H + \frac{2}{r} = \frac{2}{r} - \frac{1}{2a}.$$

We put

$$t_1 = -\epsilon a^{\frac{3}{2}}, \text{ so that } u = 0 \text{ when } t = t_1$$

Then

$$L_{t=t_1} = \frac{2}{a(1-e)} - \frac{1}{2a}.$$

Hence using the formula

$$C_b = -\left[\sum_{i=1}^{i=n} p_i \frac{dq_i}{db} + L\frac{dt_1}{db}\right]_{t=t_1} + \frac{dA}{db}$$

we see that, if we assume A to be independent of g, α, ϕ,

$$C_g = -\sqrt{a(1-e^2)}; \quad C_\alpha = -\sqrt{a(1-e^2)} \cdot \cos\phi; \quad C_\phi = 0.$$

We have still to find C_a, C_e, C_ϵ.

Since

$$\left.\begin{aligned} X &= a(\cos u - e), \\ Y &= a\sqrt{1-e^2}\sin u, \\ a^{-\frac{3}{2}}t + \epsilon &= u - e\sin u \end{aligned}\right\}$$

$$(X)_{u=0} = a(1-e), \quad (Y)_{u=0} = 0$$

$$\left(\frac{dX}{du}\right)_{u=0} = 0, \quad \left(\frac{dY}{du}\right)_{u=0} = a\sqrt{1-e^2}$$

$$(\dot{X})_{u=0} = 0, \text{ and}$$

$$\left(\dot{Y}\frac{dY}{db}\right)_{u=0} = \left[\left(\frac{dY}{du}\right)^2 \frac{du}{dt}\cdot\frac{du}{db}\right]_{u=0} = \sqrt{a}(1+e)\left(\frac{du}{db}\right)_{u=0}$$

Hence

$$C_b = -\left(\frac{2}{a(1-e)} - \frac{1}{2a}\right)\frac{dt_1}{db} - \sqrt{a}(1+e)\left(\frac{du}{db}\right)_{\substack{u=0\\t=t_1}} + \frac{dA}{db}$$

$$C_a = \frac{3}{2}\epsilon\frac{1}{\sqrt{a}}\left(\frac{2}{1-e} - \frac{1}{2}\right) - \frac{3}{2}\frac{\epsilon}{\sqrt{a}}\frac{1+e}{1-e} + \frac{dA}{da} = \frac{3}{4}\frac{\epsilon}{\sqrt{a}} + \frac{dA}{da}.$$

$$C_e = \frac{dA}{de}$$

$$C_\epsilon = \sqrt{a}\left(\frac{2}{1-e} - \frac{1}{2}\right) - \sqrt{a}\frac{1+e}{1-e} + \frac{dA}{d\epsilon} = \frac{\sqrt{a}}{2} + \frac{dA}{d\epsilon}.$$

If we put $A = -\frac{1}{2}\sqrt{a}\cdot\epsilon$, we see that

$$C_a = \frac{1}{2}\frac{\epsilon}{\sqrt{a}}, \quad C_e = 0, \quad C_\epsilon = 0,$$

$$C_g = -\sqrt{a(1-e^2)}, \quad C_\alpha = -\sqrt{a(1-e^2)}\cos\phi, \quad C_\phi = 0.$$

Hence from equation (9), the new equations are

$$\left.\begin{aligned}
\frac{d\Omega}{da} &= \frac{1}{2}a^{-\frac{1}{2}}\dot{\epsilon} + \frac{1}{2}a^{-\frac{1}{2}}\sqrt{1-e^2}(\dot{g}+\dot{\alpha}\cos\phi)\\
\frac{d\Omega}{de} &= -a^{\frac{1}{2}}\frac{e}{\sqrt{1-e^2}}(\dot{g}+\dot{\alpha}\cos\phi)\\
\frac{d\Omega}{d\epsilon} &= \frac{1}{2}a^{-\frac{1}{2}}\dot{a}\\
\frac{d\Omega}{dg} &= -\frac{1}{2}a^{-\frac{1}{2}}\sqrt{1-e^2}\,.\,\dot{a} + a^{\frac{1}{2}}\frac{e}{\sqrt{1-e^2}}\,.\,\dot{e}\\
\frac{d\Omega}{da} &= -\frac{1}{2}a^{-\frac{1}{2}}\sqrt{1-e^2}\,.\,\dot{a}\cos\phi + a^{\frac{1}{2}}\frac{e}{\sqrt{1-e^2}}\dot{e}\cos\phi + \sqrt{a(1-e^2)}\sin\phi\\
\frac{d\Omega}{d\phi} &= -\sqrt{a(1-e^2)}\,.\,\sin\phi\,.\,\dot{\alpha}.
\end{aligned}\right\}$$

§ 3. *A Method of Successive Approximation.*

In proceeding to consider consecutive terms of the disturbing function, it is assumed that a solution containing $2n$ arbitrary constants $\beta_1, \beta_2, \ldots \beta_{2n}$ has been found of the equations

$$\frac{d\Omega_1}{da_s} = \dot{C}_s - \sum_{r=1}^{r=2n}\frac{dC_r}{da_s}\dot{a}_r, \quad (s = 1, 2, \ldots 2n) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and it is required to find the equations for $\dot{\beta}_1, \dot{\beta}_2, \ldots \dot{\beta}_{2n}$ when Ω_2 is added to Ω_1.

Let

$$L_1 = \Omega_1 + \sum_{r=1}^{r=2n} C_r\,\dot{a}_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\begin{aligned}
\frac{dL_1}{d\beta_k} &= \sum_{s=1}^{s=2n}\frac{d\Omega_1}{da_s}\frac{da_s}{d\beta_k} + \sum_{r=1}^{r=2n}\sum_{s=1}^{s=2n}\frac{dC_r}{da_s}\dot{a}_r\frac{da_s}{d\beta_k} + \sum_{r=1}^{r=2n}C_r\frac{d\dot{a}_1}{d\beta_k}\\
&= \sum_{s=1}^{s=2n}\left[\dot{C}_s\frac{da_s}{d\beta_k} + C_s\frac{d\dot{a}_s}{d\beta_k}\right], \text{ by (10)}\\
&= \frac{d}{dt}\left(\sum_{s=1}^{s=2n}C_s\frac{da}{d\beta_k}\right)
\end{aligned}$$

the terms in the bracket being expressed in terms of $\beta_1, \beta_2, \ldots \beta_{2n}, t$, and $\dot{C}_s, \dot{a}_s$ being written in the above for the values of

$$\frac{dC_s}{dt}, \quad \frac{da_s}{dt}$$

when C_s, a_s are expressed in terms of $\beta_1, \beta_2, \ldots \beta_{2n}, t$.

It follows that

$$\sum_{s=1}^{s=2n}C_s\frac{da_s}{d\beta_k} = \int_{t_1}^{t}\frac{dL_1}{d\beta_k}dt + D_k,$$

$\frac{d\mathrm{L}_1}{d\beta_k}$ being expressed in terms of $\beta_1, \beta_2, \ldots \beta_{2n}, t$, and integrated with regard to t explicitly ; D_k is a function of $\beta_1, \beta_2, \ldots \beta_{2n}$, but does not contain t explicitly ; t_1 may be any function of $\beta_1, \beta_2, \ldots \beta_{2n}$, but must not contain t.

If

$$\mathrm{D}_k + (\mathrm{L}_1)_{t=t_1} \frac{dt_1}{d\beta_k} + \mathrm{E}_k = 0$$

then

$$\left.\begin{aligned} \sum_{s=1}^{s=2n} \left(\mathrm{C}_s \frac{d\alpha_s}{d\beta_k} \right) &= \frac{d}{d\beta_k} \int_{t_1}^{t} \mathrm{L}_1 \, dt - \mathrm{E}_k \\ \text{similarly} \quad \sum_{s=1}^{s=2n} \left(\mathrm{C}_s \frac{d\alpha_s}{d\beta_j} \right) &= \frac{d}{d\beta_j} \int_{t_1}^{t} \mathrm{L}_1 \, dt - \mathrm{E}_j \end{aligned}\right\} \quad \ldots \quad (12)$$

Hence

$$[\beta_k, \beta_j] = \frac{d\mathrm{E}_j}{d\beta_k} - \frac{d\mathrm{E}_k}{d\beta_j} \quad \ldots \quad (13)$$

where

$$[\beta_k, \beta_j] \equiv \sum_{s=1}^{s=2n} \left(\frac{d\mathrm{C}_s}{d\beta_j} \frac{d\alpha_s}{d\beta_k} - \frac{d\mathrm{C}_s}{d\beta_k} \cdot \frac{d\alpha_s}{d\beta_j} \right) \quad \ldots \quad (14)$$

Since E_j, E_k are independent of t explicitly, it follows that $[\beta_k, \beta_j]$ does not contain t explicitly, when expressed in terms of $\beta_1, \beta_2, \ldots \beta_{2n}$. It follows, also, that, as far as regards the determination of E_k, we can assign any value we please to t in equation (12).

If

$$\mathrm{F}_k = \mathrm{E}_k + \frac{d\mathrm{B}}{d\beta_k} \quad \ldots \quad (15)$$

when B is any arbitrary function of $\beta_1, \beta_2, \ldots \beta_{2n}$, it is clear that

$$[\beta_k, \beta_j] = \frac{d\mathrm{F}_j}{d\beta_k} - \frac{d\mathrm{F}_k}{d\beta_j} \quad \ldots \quad (16)$$

and if we put $t = t_1$ in equation (12)

$$\mathrm{F}_k = -\left[\sum_{s=1}^{s=2n} \left(\mathrm{C}_s \frac{d\alpha_s}{d\beta_k} \right) + \mathrm{L}_1 \frac{dt_1}{d\beta_k} \right]_{t=t} + \frac{d\mathrm{B}}{d\beta_k} \quad \ldots \quad (17)$$

Having thus determined $[\beta_k, \beta_j]$ we can now proceed to find the new equations.

Let the original equations be

$$\frac{d(\Omega_1 + \Omega_2)}{d\alpha_s} = \dot{\mathrm{C}}_s - \sum_{r=1}^{r=2n} \frac{d\mathrm{C}_r}{d\alpha_s} \dot{\alpha}_r, \ (s = 1, \ldots 2n) \quad \ldots \quad (18)$$

Then, since $\beta_1, \beta_2, \ldots \beta_{2n}$ have been so chosen that when $\alpha_1, \alpha_2, \ldots \alpha_{2n}, C_1, C_2, \ldots C_{2n}$, are expressed in terms of them and t

$$\frac{d\Omega_1}{d\alpha_s} = \frac{dC_s}{dt} - \sum_{r=1}^{r=2n} \frac{dC_r}{d\alpha_s} \cdot \frac{d\alpha_r}{dt}, \quad (s = 1, 2, \ldots 2n)$$

it follows that

$$\frac{d\Omega_2}{d\alpha_s} = \sum_{k=1}^{k=2n} \frac{dC_s}{d\beta_k} \dot{\beta}_k - \sum_{k=1}^{k=2n} \sum_{r=1}^{r=2n} \frac{dC_r}{d\alpha_s} \cdot \frac{d\alpha_r}{d\beta_k} \cdot \dot{\beta}_k.$$

Multiplying this equation by

$$\frac{d\alpha_s}{d\beta_j} \quad (s = 1, 2, \ldots 2n)$$

and adding the complete set we see that

$$\begin{aligned}
\frac{d\Omega_2}{d\beta_j} &= \sum_{k=1}^{k=2n} \sum_{s=1}^{s=2n} \frac{dC_s}{d\beta_k} \cdot \frac{d\alpha_s}{d\beta_j} \dot{\beta}_k \\
&\quad - \sum_{k=1}^{k=2n} \sum_{r=1}^{r=2n} \sum_{s=1}^{s=2n} \frac{dC_r}{d\alpha_s} \cdot \frac{d\alpha_s}{d\beta_j} \cdot \frac{d\alpha_r}{d\beta_k} \cdot \dot{\beta}_k \\
&= \sum_{k=1}^{k=2n} \sum_{s=1}^{s=2n} \frac{dC_s}{d\beta_k} \frac{d\alpha_s}{d\beta_j} \dot{\beta}_k \\
&\quad - \sum_{k=1}^{k=2n} \sum_{r=1}^{r=2n} \frac{dC_r}{d\beta_j} \frac{d\alpha_r}{d\beta_k} \dot{\beta}_k \\
&= \sum_{k=1}^{k=2n} [\beta_k, \beta_j] \dot{\beta}_k \\
&= \sum_{k=1}^{k=2n} \left(\frac{dF_j}{d\beta_k} - \frac{dF_k}{d\beta_j} \right) \dot{\beta}_k \\
\therefore \frac{d\Omega_2}{d\beta} &= \dot{F}_j - \sum_{k=1}^{k=2n} \frac{dF_k}{d\beta_j} \dot{\beta}_k, \quad (j = 1, 2, \ldots 2n) \quad . \quad . \quad . \quad . \quad (19)
\end{aligned}$$

are the required equations.

It is clear that that this theorem includes that of §1 as a particular case. If it is possible to express $\beta_1, \beta_2, \ldots \beta_{2n}$ in terms of $F_1, F_2, \ldots F_{2n}$, we see, by multiplying equation (19) by

$$\frac{d\beta_j}{dF_m}$$

and adding, that

$$-\frac{d\Omega_2}{dF_m} = \dot{\beta}_m - \sum_{k=1}^{k=2n} \frac{d\beta_k}{dF_m} \cdot \dot{F}_k, \quad (m = 1, 2 \ldots 2n) \quad . \quad . \quad . \quad . \quad (20)$$

There is a species of reciprocity between the set of variables $(F_1, F_2, \ldots F_{2n})$ and the set $(\beta_1, \beta_2, \ldots \beta_{2n})$.

From a consideration of the coefficient of $\dot{F}_s$ in (20), it is clear that

$$[F_r, F_s] = -\frac{d\beta_s}{dF_r} + \frac{d\beta_r}{dF_s}.$$

§ 4. If n of the functions denoted by C, say C_{n+1}, C_{n+2}, . . . C_{2n}, are functions of the n a's only, which do not correspond to them, viz. $a_1, a_2, \ldots a_n$, it is possible to indicate a method of solution of the equations :

$$\frac{d\Omega}{da_s} = \dot{C}_s - \sum_{r=1}^{r=2n} \frac{dC_r}{da_s} \cdot \dot{a}_r, \ (s = 1, 2, \ldots 2n) \quad . \quad . \quad . \quad . \quad (9)$$

which is similar to Jacobi's method of solution of the canonical equations.

Let $\beta_1, \beta_2, \ldots \beta_n, \gamma_1, \gamma_2, \ldots \gamma_n$, be $2n$ variables connected with $a_1, a_2, \ldots a_{2n}$ by the equations

$$\left.\begin{aligned} \frac{d\psi}{da_i} &= C_i - \sum_{r=n+1}^{r=2n} a_r \frac{dC_r}{da_i} \\ \frac{d\psi}{d\beta_i} &= \gamma_i \end{aligned}\right\} (i = 1, 2, \ldots n) \quad . \quad . \quad . \quad . \quad (21)$$

where ψ is a function of $a_1, a_2, \ldots a_n, \beta_1, \beta_2, \ldots \beta_n, t$ only.

Let δ denote an arbitrary variation of the variables.

Then since

$$\delta C_r = \sum_{i=1}^{i=n} \frac{dC_r}{da_i} \delta a_i + \frac{dC_r}{dt} \delta t$$

if C_r do not contain $a_{n+1}, a_{n+2}, \ldots a_{2n}$; where $r = n+1, n+2, \ldots 2n$.

$$\begin{aligned} \delta\psi &= \sum_{i=1}^{i=n} \left(\frac{d\psi}{da_i} \delta a_i + \frac{d\psi}{d\beta_i} \delta\beta \right) + \frac{d\psi}{dt} \delta t \\ &= \sum_{i=1}^{i=n} (C_i \ \delta a_i + \gamma_i \delta\beta_i) - \sum_{i=n+1}^{i=2n} a_i \delta \ C_i + \phi\delta t \end{aligned}$$

when

$$\phi \equiv \frac{d\psi}{dt} + \sum_{i=n+1}^{i=2n} a_i \frac{dC}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Let Δ denote another independent arbitrary variation.

Then

$$\begin{aligned} \Delta\delta\psi = &\sum_{i=1}^{i=n} [\Delta C_i \delta a_i + \Delta\gamma_i \delta\beta_i + C_i \Delta\delta a_i + \gamma_i \Delta\delta\beta_i] \\ &- \sum_{i=n+1}^{i=2n} (\Delta a_i \delta C_i + a_i \Delta\delta C_i) + \Delta\phi \ . \ \delta t + \phi \ . \ \Delta\delta t \end{aligned}$$

Similarly

$$\begin{aligned} \delta\Delta\psi = &\sum_{i=1}^{i=n} [\delta C_i \Delta a_i + \delta\gamma_i \Delta\beta_i + C \ \delta\Delta a_i + \gamma_i \delta\Delta\beta_i] \\ &- \sum_{i=n+1}^{i=2n} [\delta a_i \Delta C_i + a_i \delta\Delta C_i] + \delta\phi \ . \ \Delta t + \phi \ . \ \delta\Delta t. \end{aligned}$$

But

$$\Delta\delta\psi = \delta\Delta\psi,\ \delta\Delta\alpha_i = \Delta\delta\alpha_i,\ \&c.$$

Hence

$$\sum_{i=1}^{i=2n}[\delta\alpha_i\Delta C_i - \Delta\alpha_i\delta C_i] + \sum_{r=1}^{r=n}[-\delta\gamma_r\Delta\beta_r + \Delta\gamma_r\delta\beta_r] + \Delta\phi\ .\ \delta t - \delta\phi\ .\ \Delta t = 0\ . \quad (23)$$

Now $\alpha_1, \alpha_2, \ldots \alpha_{2n}$ can be expressed in terms of $\beta_1, \beta_2, \ldots \beta_n, \gamma_1, \gamma_2, \ldots \gamma_n, t$, and *vice versâ*.

Let $(\ )'$ denote that the quantity enclosed in the bracket is expressed in terms of $\beta_1, \beta_2, \ldots \beta_n, \gamma_1, \gamma_2, \gamma_n, t$; and let all quantities except ψ, which are not enclosed in $(\ )'$, be in future expressed in terms of $\alpha_1, \alpha_2, \ldots \alpha_{2n}, t$; but let ψ be in all cases expressed in terms of $\alpha_1, \alpha_2, \ldots \alpha_n, \beta_1\ \beta_2, \ldots \beta_n, t$.

Then

$$\Delta\alpha_i = \sum_{r=1}^{r=n}\left[\frac{d(\alpha_i)'}{d\beta_r}\Delta\beta_r + \frac{d(\alpha_i)'}{d\gamma_r}\Delta\gamma_r\right] + \frac{d(\alpha_i)'}{dt}\Delta t$$

$$\delta\beta_r = \sum_{i=1}^{i=2n}\left[\frac{d\beta_r}{d\alpha_i}\ \delta\alpha_i\right] + \frac{d\beta_r}{dt}\delta t,\ \&c.$$

Equation (23) can then be written

$$\begin{aligned}
&\sum_{i=1}^{i=2n}\left[\delta\alpha_i\left\{\sum_{r=1}^{r=n}\left(\frac{d(C_i)'}{d\beta_r}\Delta\beta_r + \frac{d(C_i)'}{d\gamma_r}\Delta\gamma_r\right) + \frac{d(C_i)'}{dt}\Delta t\right\}\right.\\
&-\sum_{s=1}^{s=2n}\left[\sum_{r=1}^{r=n}\left\{\frac{d(\alpha_s)'}{d\beta_r}\Delta\beta_r + \frac{d(\alpha_s)'}{\delta\gamma_r}\Delta\gamma_r\right\} + \frac{d(\alpha_s)'}{dt}\Delta t\right]\cdot\left[\sum_{i=1}^{i=n}\frac{dC_s}{d\alpha_i}\delta\alpha_i + \frac{dC_s}{dt}\delta t\right]\\
&+\sum_{r=1}^{r=n}\left[\Delta\gamma_r\left\{\sum_{i=1}^{i=2n}\frac{d\beta_r}{d\alpha_i}\delta\alpha_i + \frac{d\beta_r}{dt}\delta t\right\}\right.\\
&\left.-\Delta\beta_r\left\{\sum_{i=1}^{i=2n}\frac{d\gamma_r}{d\alpha_i}\delta\alpha_i + \frac{d\gamma_r}{dt}\delta t\right\}\right]\\
&+\delta t\left\{\sum_{r=1}^{r=n}\left(\frac{d(\phi)'}{d\beta_r}\Delta\beta_r + \frac{d(\phi)'}{d\gamma_r}\Delta\gamma\right) + \frac{d(\phi)'}{dt}\Delta t\right\}\\
&-\Delta t\left\{\sum_{i=1}^{i=2n}\frac{d\phi}{d\alpha_i}\delta\alpha_i + \frac{d\phi}{dt}\delta t\right\} = 0\ \ldots\ldots\ldots\ldots\ldots\ (24)
\end{aligned}$$

But the variations

$$\delta\alpha_1, \delta\alpha_2, \ldots \delta\alpha_{2n}, \delta t, \Delta\beta_1, \Delta\beta_2, \ldots \Delta\beta_n, \Delta\gamma_1, \Delta\gamma_2, \ldots \Delta\gamma_n, \Delta t$$

are all arbitrary and independent; each coefficient of these quantities in equation (24) must therefore be zero.

Hence we find :

$$\left.\begin{aligned}\frac{d(\mathrm{C}_i)'}{d\beta_r}-\sum_{s=1}^{s=2n}\frac{d(\alpha_s)'}{d\beta_r}\cdot\frac{d\mathrm{C}_s}{d\alpha_i}-\frac{d\gamma_r}{d\alpha i}&=0\\ \frac{d(\mathrm{C}_i)'}{d\gamma_r}-\sum_{s=1}^{s=2n}\frac{d(\alpha_s)'}{d\gamma_r}\cdot\frac{d\mathrm{C}_s}{d\alpha_i}+\frac{d\beta_r}{d\alpha_i}&=0\end{aligned}\right\}\quad\ldots\ldots\quad(25)$$

$$\frac{d(\mathrm{C}_i)'}{dt}-\sum_{s=1}^{s=2n}\frac{d(\alpha_s)'}{dt}\cdot\frac{d\mathrm{C}_s}{d\alpha_i}-\frac{d\phi}{d\alpha_i}=0\quad\ldots\ldots\quad(26)$$

$$\left.\begin{aligned}-\frac{d\gamma_r}{dt}-\sum_{s=1}^{s=2n}\frac{d(\alpha_s)'}{d\beta_r}\cdot\frac{d\mathrm{C}_s}{dt}+\frac{d(\phi)'}{d\beta_r}&=0\\ \frac{d\beta_r}{\delta t}-\sum_{s=1}^{s=2n}\frac{d(\alpha_s)'}{d\gamma_r}\cdot\frac{d\mathrm{C}_s}{dt}+\frac{d(\phi)'}{d\gamma_r}&=0\end{aligned}\right\}\quad\ldots\ldots\quad(27)$$

$$\frac{d(\phi)'}{dt}-\sum_{s=1}^{s=2n}\frac{d(\alpha_s)'}{dt}\cdot\frac{d\mathrm{C}_s}{dt}-\frac{d\phi}{dt}=0\quad\ldots\ldots\quad(28)$$

Now the original equations are

$$\frac{d\Omega}{d\alpha_s}=\dot{\mathrm{C}}_s-\sum_{r=1}^{r=2n}\frac{d\mathrm{C}_r}{d\alpha_s}\dot{\alpha}_r,\ (s=1,\ 2,\ \ldots\ 2n)\quad\ldots\ldots\quad(9)$$

Hence

$$\begin{aligned}\frac{d(\Omega)'}{d\beta_r}&=\sum_{s=1}^{s=2n}\left[\frac{d(\alpha_s)'}{d\beta_r}\cdot\dot{\mathrm{C}}_s-\sum_{i=1}^{i=2n}\frac{d\mathrm{C}_i}{d\alpha_s}\cdot\frac{d(\alpha_s)'}{d\beta_r}\cdot\dot{\alpha}_i\right]\\ &=\sum_{s=1}^{s=2n}\left[\frac{d(\alpha_s)'}{d\beta_r}\cdot\dot{\mathrm{C}}_s\right]-\sum_{i=1}^{i=2n}\dot{\alpha}_i\frac{d(\mathrm{C}_i)'}{d\beta_r}\\ &=\sum_{s=1}^{s=2n}\left[\frac{d(\alpha_s)'}{d\beta_r}\cdot\dot{\mathrm{C}}_s-\frac{d(\mathrm{C}_s)'}{d\beta_r}\cdot\dot{\alpha}_s\right]\\ &=\sum_{s=1}^{s=2n}\left[\frac{d\mathrm{C}_s}{dt}\cdot\frac{d(\alpha_s)'}{d\beta_r}+\sum_{i=1}^{i=2n}\frac{d\mathrm{C}_s}{d\alpha_i}\dot{\alpha}_s\frac{d(\alpha_s)'}{d\beta_r}\right]\\ &\quad-\sum_{i=1}^{i=2n}\left[\dot{\alpha}_i\left\{\sum_{s=1}^{s=2n}\frac{d(\alpha_s)'}{d\beta_r}\cdot\frac{d\mathrm{C}_s}{d\alpha_i}+\frac{d\gamma_r}{d\alpha_i}\right\}\right]\ \text{by (25)}\\ &=-\frac{d\gamma_r}{dt}+\frac{d(\phi)'}{d\beta_r}-\sum_{i=1}^{i=2n}\dot{\alpha}_i\frac{d\gamma_r}{d\alpha_i},\ \text{by (27)}\end{aligned}$$

whence we obtain

$$\left.\begin{aligned}\dot{\gamma}_r&=-\frac{d}{d\beta_r}(\phi-\Omega)'\\ \text{Similarly}\qquad\dot{\beta}_r&=\frac{d}{d\gamma_r}(\phi-\Omega)'\end{aligned}\right\}\ (r=1,\ 2,\ \ldots\ n)\quad\ldots\quad(29)$$

These new equations are canonical.

If $\phi=\Omega$, we see that $\beta_1,\ \beta_2,\ \ldots\ \beta_n,\ \gamma_1,\ \gamma_2,\ \ldots\ \gamma_n$, form a

canonical set of arbitrary constants, the solution being given by the equations

$$\left.\begin{aligned}\frac{d\psi}{d\alpha_i} &= C_i - \sum_{r=n+1}^{r=2n} \alpha_r \frac{dC_r}{d\alpha_i} \\ \frac{d\psi}{d\beta_i} &= \gamma_i\end{aligned}\right\} \quad (i = 1, 2, \ldots n) \quad . \quad . \quad . \quad . \quad (21)$$

when

$$\phi = \frac{d\psi}{dt} + \sum_{i=n+1}^{i=2n} \alpha_i \frac{dC_i}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

From the equations

$$\frac{d\psi}{d\alpha_i} = C_i - \sum_{r=n+1}^{r=2n} \alpha_r \frac{dC_r}{d\alpha_i}, \quad (i = 1, 2, \ldots n)$$

we can in general find α_{n+1}, α_{n+2}, . . α_{2n} in terms of

$$\alpha_1, \alpha_2, \ldots \alpha_n, \frac{d\psi}{d\alpha_1}, \frac{d\psi}{d\alpha_2}, \ldots \frac{d\psi}{d\alpha_n}, t.$$

If these values are substituted in the equation

$$\frac{d\psi}{dt} + \sum_{i=1}^{i=n} \alpha_i \frac{dC_i}{dt} = \Omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

we obtain a differential equation of the first order, for ψ in terms of $\alpha_1, \alpha_2, \ldots \alpha_n, t$. If any complete solution of this equation, containing n arbitrary constant $\beta_1, \beta_2, \ldots \beta_n$ be found, the solution of the equations (9) is given by equations (21).

If, now, a further term Ω_1 is added to Ω, the equations for the variations of the arbitrary constants are (by [29]) seen to be

$$\left.\begin{aligned}\dot{\gamma}_r &= \frac{d\Omega_1}{d\beta_r} \\ -\dot{\beta}_r &= \frac{d\Omega_1}{d\gamma_r}\end{aligned}\right\} (r = 1, 2, \ldots n) \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

which form a canonical set.

In particular, if $C_{n+1}, C_{n+2}, \ldots C_{2n}$, are all zero or definite constants, we see, by putting

$$\psi = \sum_{i=1}^{i=n} \alpha_i \beta_i$$

in which $\phi = 0$, that $\alpha_1, \alpha_2, \ldots \alpha_n, C_1, C_2, \ldots C_n$, in this case form a canonical set of variables. For example, in the case of the planetary theory,

$$a, \qquad g, \qquad \alpha$$

$$\tfrac{1}{2}\frac{\epsilon}{\sqrt{a}}, \quad -\sqrt{a(1-e^2)}, \quad -\sqrt{a(1-e^2)} \cdot \cos\phi$$

form a canonical set of arbitrary constants of the elliptic motion.

Note on Mr. A. Y. G. Campbell's paper "On the Variation of Uncanonical Arbitrary Constants, with an Application to the Planetary Theory." By Sir Robert S. Ball, LL.D., F.R.S.

The object of this note is to set down another method by which the calculation of Lagrange's "parentheses" in the planetary theory can be effected very simply. The method was suggested by a manuscript which Mr. Campbell kindly communicated to me. I have also to acknowledge the assistance derived from the instructive remarks of Mr. Cowell and Mr. Whittaker, to whom, as well as to Mr. Campbell, it has been my privilege to have lectured at Cambridge.

Adopting the notation of Mr. Campbell's manuscript, we put

$$\frac{\partial(x, \dot{x})}{\partial(\lambda, \mu)} \text{ for } \frac{dx}{d\lambda} \cdot \frac{d\dot{x}}{d\mu} - \frac{dx}{d\mu} \cdot \frac{d\dot{x}}{d\lambda},$$

where λ, μ are any two of the six arbitrary constants.

We thus have the expression for the "parenthesis"

$$[\lambda, \mu] = \frac{\partial(x, \dot{x})}{\partial(\lambda, \mu)} + \frac{\partial(y, \dot{y})}{\partial(\lambda, \mu)} + \frac{\partial(z, \dot{z})}{\partial(\lambda, \mu)}.$$

We now make the transformation to another set of rectangular coordinates x_1, y_1, z_1, where

$$\begin{aligned} x &= l_1 x_1 + l_2 y_1 + l_3 z_1 \\ y &= m_1 x_1 + m_2 y_1 + m_3 z_1 \\ z &= n_1 x_1 + n_2 y_1 + n_3 z_1 \end{aligned}$$

It is unnecessary to write down the general result. For the planetary theory we take the special case where

$$\begin{array}{lll} l_1 = \cos\alpha & l_2 = -\sin\alpha\cos\phi & l_3 = 0 \\ m_1 = \sin\alpha & m_2 = \cos\alpha\cos\phi & m_3 = 0 \\ n_1 = 0 & n_2 = \sin\phi & n_3 = 0 \end{array}$$

And if we express x_1 by ξ and y_1 by η we get after a little reduction

$$[\lambda, \mu] = \frac{\partial(\xi, \dot{\xi})}{\partial(\lambda, \mu)} + \frac{\partial(\eta, \dot{\eta})}{\partial(\lambda, \mu)} - \sin\phi\,(\xi\dot{\eta} - \dot{\xi}\eta)\frac{\partial(\alpha, \phi)}{\partial(\lambda, \mu)}$$

$$+ \cos\phi \frac{d\alpha}{d\lambda}\frac{d}{d\mu}(\xi\dot{\eta} - \dot{\xi}\eta) - \cos\phi \frac{d\alpha}{d\mu}\frac{d}{d\lambda}(\xi\dot{\eta} - \dot{\xi}\eta)$$

We now transform to two new variables X and Y given by the equations

$$\begin{aligned} \xi &= X\cos g - Y\sin g \\ \eta &= X\sin g + Y\cos g \end{aligned}$$

Also from the known properties of elliptic motion

$$\xi\dot{\eta} - \eta\dot{\xi} = \sqrt{a(1-e^2)} = h,$$

whence the general symbolic representation of a "parenthesis"

$$[\lambda, \mu] = \frac{\partial(X, \dot{X})}{\partial(\lambda, \mu)} + \frac{\partial(Y, \dot{Y})}{\partial\lambda, \mu)} - h \sin\phi \frac{\partial(\alpha, \phi)}{\partial(\lambda, \mu)} + \cos\phi \frac{\partial(\alpha, h)}{\partial(\lambda, \mu)} + \frac{\partial(g, h)}{\partial(\lambda, \mu)}.$$

It will now be convenient to divide the fifteen "parentheses" into two groups, those which contain ϵ and those which do not. Taking the former first, we have

$$[\epsilon, \mu] = \frac{\partial(X, \dot{X})}{\partial(\epsilon, \mu)} + \frac{\partial(Y, \dot{Y})}{\partial(\epsilon, \mu)},$$

for obviously the three latter terms in the general expression do not contribute.

As ϵ only occurs in the expression $a^{-\frac{3}{2}}t + \epsilon$, and as t is never found except in this expression

$$\frac{dX}{d\epsilon} = a^{\frac{3}{2}}\dot{X} \qquad \frac{dY}{d\epsilon} = a^{\frac{3}{2}}\dot{Y}$$

$$\frac{d\dot{X}}{d\epsilon} = a^{\frac{3}{2}}\ddot{X} \qquad \frac{d\dot{Y}}{d\epsilon} = a^{\frac{3}{2}}\ddot{Y}$$

whence we easily find

$$\frac{d\dot{X}}{d\epsilon} = -\frac{a^{\frac{3}{2}}}{\rho^3}X; \quad \frac{d\dot{Y}}{d\epsilon} = -\frac{a^{\frac{3}{2}}}{\rho^3}Y$$

We thus have

$$[\epsilon, \mu] = \frac{1}{2}a^{\frac{3}{2}}\frac{d}{d\mu}(\dot{X}^2 + \dot{Y}^2) + \frac{1}{2}a^{\frac{3}{2}}\frac{1}{\rho^3}\frac{d}{d\mu}(X^2 + Y^2) = \frac{1}{2}a^{-\frac{1}{2}}\frac{da}{d\mu}.$$

For those parentheses $[\lambda, \mu]$ whereof neither is ϵ the following considerations will simplify the result.

In the values of X, $\dot{X}$, Y, $\dot{Y}$, t and ϵ enter only by powers of the exponential $e^{i(a^{-\frac{3}{2}}t+\epsilon)}$, where e is the base of natural logs. If this be differentiated with respect to constants which are neither a nor ϵ, this exponential is unaffected. But we know that the time disappears from the parenthesis; therefore it is immaterial what be the value of $a^{-\frac{3}{2}}t + \epsilon$, and we may accordingly take both t and ϵ zero. If, however, the exponential be differentiated with respect to a, then we have terms like $te^{i(a^{-\frac{3}{2}}t+\epsilon)}$. But such terms must be mutually destructive, for the exponential terms simply can never annul them. We need therefore never have formed such terms. Hence we may *before* differentiation make both t and ϵ zero. Hence both $\dot{X}$ and $\dot{Y}$ may be treated as non-existent, and

$$\frac{\partial(X, \dot{X})}{\partial(\lambda, \mu)} + \frac{\partial(Y, \dot{Y})}{\partial(\lambda, \mu)}$$

will be zero whenever ϵ is neither λ nor μ.

We have thus reduced the formulæ for the parentheses to the simple forms following :—

$$[\epsilon, \mu] = \frac{1}{2}\, a^{-\frac{1}{2}} \frac{da}{d\mu} \text{ for all which contain } \epsilon$$

$$[g, \mu] = \frac{\partial(g, h)}{\partial(g, \mu)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad g$$

$$[\phi, \mu] = h \sin \phi \frac{\partial(\phi, a)}{\partial(\phi, \mu)} \quad . \quad . \quad . \quad . \quad . \quad \phi$$

$$[\lambda, \mu] = \cos\phi \frac{\partial(a, h)}{\partial(\lambda, \mu)} \text{ for all which contain only } a, e, a$$

Perhaps the most instructive form is that which comprises *all* the fifteen parentheses in the general expression

$$[\lambda, \mu] = \tfrac{1}{2} a^{-\frac{1}{2}} \frac{\partial(\epsilon, a)}{\partial(\lambda, \mu)} + \frac{\partial(g, h)}{\partial(\lambda, \mu)} + \cos\phi \frac{\partial(a, h)}{\partial(\lambda, \mu)} - h \sin \phi \frac{\partial(a, \phi)}{\partial(\lambda, \mu)}.$$

We thus obtain by inspection

$$[\epsilon, a] = \frac{1}{2}\, a^{-\frac{1}{2}} \qquad [\epsilon, e] = 0 \qquad [\epsilon, a] = 0 \qquad [\epsilon, \phi] = 0 \qquad [\epsilon, g] = 0$$

$$[g, a] = \frac{dh}{da} \qquad [g, e] = \frac{dh}{de} \qquad [g, a] = 0 \qquad [g, \phi] = 0$$

$$[\phi, a] = 0 \qquad [\phi, e] = 0 \qquad [\phi, a] = h \sin \phi$$

$$[e, a] = 0 \qquad [a, a] = \cos\phi \frac{dh}{da} \qquad [a, e] = \cos\phi \frac{dh}{de}$$

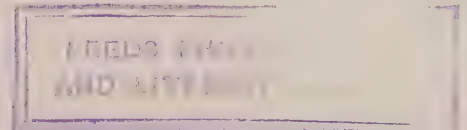
where

$$h = \sqrt{a(1 - e^2)}; \quad \frac{dh}{da} = \frac{1}{2}\, a^{-\frac{1}{2}} \sqrt{1 - e^2}; \quad \frac{dh}{de} = -\frac{a^{\frac{1}{2}} e}{\sqrt{1 - e^2}}$$

thus giving the well-known expressions in the Planetary Theory.

On Differential Refraction to Terms of Higher Orders than the First. By H. H. Turner, M.A., B.Sc., Savilian Professor.

1. For moderate zenith distances the differential refraction on, say, a photographic plate is expressed with sufficient accuracy by the first powers of the relative coordinates when the field is not too large ; but a plate may be exposed at a considerable zenith distance for the purpose of measuring parallax or some other reason, and again the field photographed may be of considerable extent. In such cases the first powers of the coordinates no longer afford a sufficiently accurate approximation, and we must take in



higher powers. Investigations of this kind have been made by Rambaut (*Ast. Nach.* 3125), Kapteyn (*Bull. Com. Perm.* T. III.), and others; but I cannot but feel that the formulæ are greatly simplified by the use of rectangular coordinates such as I have advocated in previous papers (*M.N.* liv. p. 11, liv. p. 489, lv. p. 102, lv. p. 419, &c.), and in the present paper the differential refraction will be investigated by the use of such coordinates.

2. In *M.N.* liv. p. 18 I have given the formulæ for differential refraction as far as the first order. The expression there expanded is not, however, quite accurate, and, although correct to the second order, will not do beyond that order. If O be the centre of the plate, Ox, Oy the axes, and S, Z the projections on the plate of a star and the zenith respectively, then the effect of refraction is

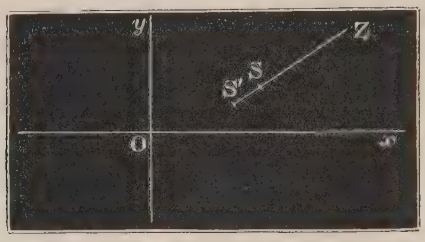


FIG. 1.

to displace S to S′ in the line S Z. So far this is rigidly accurate, since the great circle through the star and the zenith in which the refraction displacement takes place projects into the straight line S Z. But it is assumed in the paper above quoted that S S′ is proportional to tangent zenith distance, which is not quite accurate; in fact, S S′ is assumed equal to the arc of the sphere of which it is the projection; and this is only true to the second order of small quantities, *i.e.* when S is near O. We proceed to deduce the accurate formulæ.

3. Let (X, Y) be coordinates of Z; (x, y) of S; C the centre of the sphere; so that

$$x = \tan \mathrm{SCO} \cdot \cos \mathrm{SO}x, \qquad y = \tan \mathrm{SCO} \cdot \sin \mathrm{SO}x$$
$$\mathrm{X} = \tan \mathrm{ZCO} \cdot \cos \mathrm{ZO}x, \qquad \mathrm{Y} = \tan \mathrm{ZCO} \cdot \sin \mathrm{ZO}x.$$

Then the direction cosines of the line C S referred to rectangular axes at C are proportional to x, y, 1; and of C Z to X, Y, 1. Hence

$$\cos \mathrm{SCZ} = \frac{1 + x\mathrm{X} + y\mathrm{Y}}{\sqrt{1 + x^2 + y^2}\,(1 + \mathrm{X}^2 + \mathrm{Y}^2)},$$

whence we obtain

$$\tan \mathrm{SCZ} = \pm \frac{\sqrt{(x - \mathrm{X})^2 + (y - \mathrm{Y})^2 + (x\mathrm{Y} - y\mathrm{X})^2}}{1 + x\mathrm{X} + y\mathrm{Y}}.$$

Now the coordinates of S′ may be written

$$x + t(x - X), \qquad y + t(y - Y)$$

since it lies on the straight line S Z. Hence

$$\tan SCS' = \pm \frac{t\sqrt{(x-X)^2 + (y-Y)^2 + (xY - yX)^2}}{1 + x^2 + y^2 + tx(x-X) + ty(y-Y)}.$$

And

$$\tan SCS' = \mu \tan SCZ,$$

where μ is the constant of refraction, Thus :

$$\pm t(1 + xX + yY) = \mu\{1 + x^2 + y^2 + tx(x-X) + ty(y-Y)\}.$$

Since the stars are displaced *towards* the zenith it is clear that the lower sign must be used if μ be taken positive. Hence

$$t = -\mu \cdot \frac{1 + x^2 + y^2}{1 + xX + yY + \mu\{x(x-X) + y(y-Y)\}}$$

and the refractions in x and y are $t\ (x-X)$ and $t\ (y-Y)$ respectively.

In obtaining these results we have made no approximation beyond that of taking the refraction equal to its tangent, which is true to the second order of refraction.

4. We may now expand these expressions in powers of x and y. But before doing so we may reject the term in the denominator multiplied by μ. All the quantities being expressed in circular measure we have approximately

$$\mu = 1/3600 = \cdot 00028$$
$$\mu^2 = \cdot 00000008.$$

The effect of the denominator $P + \mu Q$ is to multiply the numerator by $\left(1 - \mu \frac{Q}{P}\right)\frac{1}{P}$. Now x^2 and y^2 are small quantities ; if a star be 10° from the centre of the plate, which may be adopted as a superior limit for most photographs $x^2 + y^2$ is about 0·03. Thus the largest terms introduced into the coordinates of S′ by μQ may be taken equal to

$$+ \mu^2 Q(X - x)/P^2.$$

Again it is only at great zenith distances where X and Y are large that such terms can be sensible. Hence we may neglect x and y in comparison with X and Y. Thus :

$$Q = -(Xx + Yy) \text{ and } P = Xx + Yy + 1.$$

For simplicity suppose Y=0, so that the axis of x is directed towards the zenith. Then the largest term of this kind under consideration is

$$\mu^2 X \,.\, Q/P^2 = \mu^2 X^2 \,.\, x/(1+Xx)^2.$$

Suppose the star 10° from the centre as before, so that $x = \tan 10°$. and let us put this term equal to 0″·025, or in circular measure $\frac{\pi}{180\times 60\times 60}$. Then

$$\frac{\pi}{180\times 60\times 2400} = \mu^2 X^2 \,.\, \frac{\pi}{18} \,.\, \Big/\left(1+\frac{1}{6}X^2\right)$$

or taking the square root

$$\frac{1}{1200} = \frac{1}{3600} X \Big/ \left(1+\frac{1}{6}X\right)$$

or

$$3\left(1+\frac{1}{6}X\right) \quad X.$$

Thus X=6, which corresponds to a zenith distance of 80° 32′. The term is clearly very small in all cases likely to occur in practice. Hence we may omit the term in the denominator multiplied by μ and write

$$t = -\mu \,.\, \frac{1+x^2+y^2}{1+xX+yY}$$

with the refractions in x and y

$$\Delta x = (X-x)t, \qquad \Delta y = (Y-y)t$$

$$\therefore \frac{\Delta x}{\mu} = X$$
$$-x(1+X^2)$$
$$+Xx^2(2+X^2)+Xy^2$$
$$-x^3(1+X^2)^2 - xy^2(1+X^2)$$
$$+ \&c.$$

Since X must be greater than unity to make the higher orders sensible, the largest term of the second order is clearly X^3x^2, and of the third order X^4x^3.

In $\frac{1}{\mu}\Delta y$ there are no terms of the second and third orders comparable with these. The series will not be convergent if Xx is equal to or exceeds unity; but this would mean that the lower edge of the plate is below the horizon, which does not occur in practice. If, however, the plate is near the horizon, it is clear that the convergence is slow, and the expansion in series is not suitable; but the expressions are so simple that special cases can easily be considered individually. There is no need to go into wearisome detail here.

7. We may derive these expressions in a different manner, which has some interesting features. If we project upon the

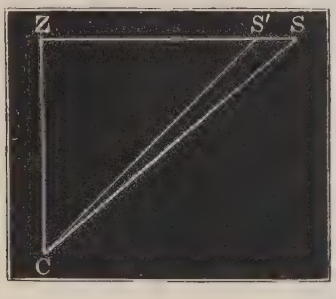


Fig. 2.

tangent-plane at the zenith, and S (fig. 2) be displaced to S′, C being the centre of the sphere and Z the zenith, we have

$$\theta = \angle \mathrm{SCS}' = \mu \tan \mathrm{ZD} = \mu \,.\, \mathrm{SZ} = \mu \,.\, r,$$

say ; and

$$r + \Delta r = \mathrm{ZS}' = \tan (\mathrm{ZCS} - \theta)$$

$$= \frac{r - \mu r}{1 + \mu r^2}.$$

Thus

$$\Delta r = \frac{-\mu r(1 + r^2)}{1 + \mu r^2}.$$

Let (u, v) be the coordinates of S on the tangent-plane at Z. Then $r^2 = u^2 + v^2$ and

$$\mathrm{CS}^2 = 1 + u^2 + v^2,$$

and

$$\Delta u = -\mathrm{SS}' \,.\, \frac{u}{\mathrm{ZS}} = -\mu u \,.\, \frac{1 + u^2 + v^2}{1 + \mu(u^2 + v^2)} = -\rho u, \text{ say}$$

$$\Delta v = -\mathrm{SS}' \,.\, \frac{v}{\mathrm{ZS}} = -\mu v \,.\, \frac{1 + u^2 + v^2}{1 + \mu(u^2 + v^2)} = -\rho v.$$

Now x and y are connected with u and v by relations of the form

$$x = \frac{au + bv + c}{1 - ku - lv}, \qquad y = \frac{du + ev + f}{1 - ku - lv},$$

where a, b, c . . . are constants (see *Monthly Notices*, vol. liv. p. 11). Thus

$$x + \Delta x = \frac{au + bv + c - \rho(au + bv)}{1 - ku - lv + \rho(ku + lv)},$$

with a similar equation for $y + \Delta y$. Multiply up and use the value of x in terms of u and v to cancel equivalent terms.

Thus

$$\Delta x\{1-ku-lv+\rho(ku+lv)\} = -\rho\{au+bv+x(ku+lv)\} \\ -\rho(c-x)$$

or on substituting

$$\rho = \frac{\mu(1+u^2+v^2)}{1+\mu(u^2+v^2)},$$

and simplifying we get

$$\Delta x = +\frac{\mu(c-x)(1+u^2+v^2)}{1-ku-lv+\mu\{(u^2+v^2)+ku+lv\}}.$$

8. There are relations between the coefficients a, b, c, &c., which may be obtained as follows. Let Z, O be the origins of the two systems of coordinates, S a star. Let the initial lines

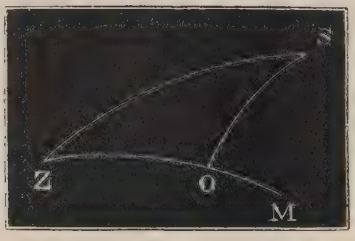


Fig. 3.

make angles α and β respectively with the line ZO; and let $ZO = \phi$. Then

$$u = \tan ZS \cos(SZO+\alpha), \qquad v = \tan ZS \sin(SZO+\alpha)$$
$$x = \tan SO \cos(SOM+\beta), \qquad v = \tan SO \sin(SOM+\beta).$$

Now

$$\sin ZO \sin ZS \cos SZO = \cos SO - \cos SZ \cos ZO$$
$$= \cos SO(1-\cos^2 ZO) + \sin SO \sin ZO \cos ZO \cos SOM$$

Thus

$$\sin ZS \cos SZO = \cos SO \sin ZO + \cos ZO \sin SO \cos SOM$$

And

$$\sin ZS \sin SZO = \sin SO \sin SOM.$$

Also

$$\cos ZS = \cos SO \cos ZO - \sin SO \sin ZO \cos SOM.$$

Dividing the two previous equations by this, we get expressions for tan ZS cos SZO and tan ZS sin SZO; *i.e.* for $u\cos\alpha+v\sin\alpha$, and $-u\sin\alpha+v\cos\alpha$. The expressions are fractions the

numerators and denominators of which may each be divided by cos SO : cos ZO ; and then $x \cos\beta + y \sin\beta$ and $-x \sin\beta + y \cos\beta$ may be substituted for tan SO cos SOM and tan SO sin SOM. We then obtain

$$u \cos\alpha + v \sin\alpha = \frac{\tan\phi + x \cos\beta + y \sin\beta}{1 - \tan\phi\,(x \cos\beta + y \sin\beta)},$$

$$-u \sin\alpha + v \cos\alpha = \frac{\{-x \sin\beta + y \cos\beta\} \sec\phi}{1 - \tan\phi\,(x \cos\beta + y \sin\beta)}.$$

and similarly

$$x \cos\beta + y \sin\beta = \frac{u \cos\alpha + v \sin\alpha - \tan\phi}{1 + \tan\phi(u \cos\alpha + v \sin\alpha)}$$

$$-x \sin\beta + y \cos\beta = \frac{\{-u \sin\alpha + v \cos\alpha\} \sec\phi}{1 + \tan\phi\,(u \cos\alpha + v \sin\alpha)}.$$

9. Thus in the notation of § 7—viz.

$$x = \frac{au + bv + c}{1 - ku - lv}, \qquad y = \frac{du + ev + f}{1 - ku - lv}$$

we have

$$a = \cos\alpha \cos\beta + \sin\alpha \sin\beta \sec\phi$$
$$b = \sin\alpha \cos\beta - \cos\alpha \sin\beta \sec\phi$$
$$c = -\tan\phi \cos\beta$$
$$d = \cos\alpha \sin\beta - \sin\alpha \cos\beta \sec\phi$$
$$e = \sin\alpha \sin\beta + \cos\alpha \cos\beta \sec\phi$$
$$f = -\tan\phi \sin\beta$$
$$k = -\tan\phi \cos\alpha$$
$$l = -\tan\phi \sin\alpha.$$

Further, we remark that the quantities (X, Y) being the values of x and y when $u=0$ and $v=0$ are

$$X = c = -\tan\phi \cos\beta, \qquad Y = f = -\tan\phi \sin\beta.$$

10. For the refraction we want the expressions $1 + u^2 + v^2$ and $1 - ku - lv$ in terms of x and y. Squaring the expressions for $u \cos\alpha + v \sin\alpha$ and $-u \sin\alpha + v \cos\alpha$, and adding we get $u^2 + v^2$, and to this we have to add unity. Perform the operation thus. Put for a moment

$$\xi = x \cos\beta + y \sin\beta, \qquad \eta = -x \sin\beta + y \cos\beta$$

and note that

$$\xi^2 + \eta^2 = x^2 + y^2$$

Then

$$1+(u\cos\alpha+v\sin\alpha)^2=1+\frac{\tan^2\phi+2\xi\tan\phi+\xi^2}{1-2\xi\tan\phi+\xi^2\tan^2\phi}$$

$$=\frac{(1+\xi^2)\sec^2\phi}{(1-\xi\tan\phi)^2}$$

$$(-u\sin\alpha+v\cos\alpha)^2=\frac{\eta^2\sec^2\phi}{(1-\xi\tan\phi)^2}$$

Thus

$$1+u^2+v^2=\frac{(1+x^2+y^2)\sec^2\phi}{(1-\xi\tan\phi)^2}$$

$$=\frac{(1+x^2+y^2)(1+\mathrm{X}^2+\mathrm{Y}^2)}{(1+x\mathrm{X}+y\mathrm{Y})^2}$$

from the expressions for X and Y given in the last paragraph.

Again

$$1-ku-lv=1+\tan\phi(u\cos\alpha+v\sin\alpha)$$

$$=1+\frac{\tan^2\phi+(x\cos\beta+y\sin\beta)\tan\phi}{1-\tan\phi(x\cos\beta+y\sin\beta)}$$

$$=\frac{1+\mathrm{X}^2+\mathrm{Y}^2}{1+x\mathrm{X}+y\mathrm{Y}}$$

11. Substituting these now in the expression of § 7—viz.

$$\Delta x=\frac{\mu(c-x)(1+u^2+v^2)}{1-ku-lv+\mu\{u^2+v^2+ku+lv\}}$$

and in the first instance neglecting the term multiplied by μ in the denominator we get

$$\Delta x=\mu\frac{(\mathrm{X}-x)(1+x^2+y^2)}{1+\mathrm{X}x+\mathrm{Y}y}$$

as in § 5. The neglected term is easily seen to be identical with that in § 3 and § 4 on remarking that the coefficient of μ^2 is

$$u^2+v^2+ku+lv\equiv(1+u^2+v^2)-(1-ku-lv)$$

$$=\frac{(1+\mathrm{X}^2+\mathrm{Y}^2)\{x(x-\mathrm{X})+y(y-\mathrm{Y})\}}{(1+x\mathrm{X}+y\mathrm{Y})^2}.$$

12. The same algebraical expression applies when instead of the mean coordinates of the star we use the apparent coordinates, modifying the value of μ accordingly.

University Observatory, Oxford:
1897 *January* 7.

The Heliographic Coordinates of Sun-spots and Faculæ on the Stonyhurst Drawings. By the Rev. A. L. Cortie, S.J.

The photographic method of registering the positions of sun-spots and faculæ, and their micrometrical measurement from the photographs as practised at Greenwich, is doubtless the most perfect of those in use. Taking the positions as reduced, and published in the annual volumes of "Spectroscopic and Photographic Results" as standard, it will be useful to compare with them some positions which have been obtained from the Stonyhurst drawings by the use of a set of orthographic projections of the parallels of latitude and meridians of longitude, and to explain the method by which these positions have been obtained. For the process of measurement by means of projections is most simple and expeditious, and, moreover, is susceptible of considerable accuracy. The drawings themselves are made by projecting the image of the Sun on to a sketching-board, the diameter of the image being $10\frac{1}{2}$ inches. It is only necessary to rule one horizontal diameter of the circle prepared to receive the solar image in order to adjust the drawing to the apparent north point. This is effected by causing a spot to run truly along this diameter by means of the slow-motion rods of the telescope. It is better to choose a small spot for this purpose.

As the heliographic latitude of the centre of the Sun's disc varies between $\pm 7°\ 15'$, eight orthographic projections of the meridians and parallels, corresponding to the fifteen positions of the pole ± 0 to $\pm 7°$, were drawn with great care and exactness by Mr. William McKeon. They were then traced on glazed linen, so as to permit of their being placed over a drawing for measurement, all the details on the drawings being easily seen through the projection. Moreover they are very durable, and, as experience has proved, not liable to shrink, thus being an excellent substitute for costly projections ruled on glass.

Each of the projections has, moreover, a scale of degrees from 0 to $\pm 30°$, ruled along its limbs north and south of the projection of the Sun's equator, to enable the disc to be adjusted for the varying position of the angle of inclination of the Sun's polar axis, this variation being within the limits $\pm 26°{\cdot}5$ from the N. point. With these discs the "Ephemeris for Physical Observations of the Sun," published yearly in the *Companion to the Observatory*, must be used, values of P, D, and L being easily interpolated for the day and time of observation.

The use of the projections will be best illustrated by an example. To find the heliographic coordinates of a sun-spot drawn in the S.F. quadrant of the Sun at $9^h\ 37^m$ G.M.T. on 1893 Aug. 9. For this date and time $P = +14°{\cdot}33$, $D = +6°{\cdot}46$, $L = 266°{\cdot}56$. Measure the apparent latitude and longitude by means of the projection for $D = \pm 6°$, adjusting it with reference to the horizontal diameter ruled on the picture to the reading

+14°·3. The readings are for latitude −16°·8, for longitude −53°·5. If the spot should be large, take the readings of its edges and deduce the mean. Since the meridians and parallels are drawn at intervals of 10°, in estimating degrees and parts of a degree it will be necessary to use subsidiary scales, allowing for the gradual foreshortening as the limbs are approached. These can be ruled on slips of paper as required.

The projection readings must now be corrected. The latitude of the centre of the disc at the time of observation was +6°·46, and we have used the projection corresponding to latitude +6°. If the spot had been situated on the central meridian it is evident that its latitude being south the correction to be applied would have been +0°·46. But it is distant 53°·5 from the central meridian. The distance between the projections of the equators on two successive discs is evidently 1° on the central meridian and 0° at the points to which they converge on or in the immediate neighbourhood of the limb. At intermediate meridians it is successively and proportionally less than 1°, reckoning from the central meridian. The factor to be applied is the natural cosine of its angular distance from the central meridian, and the correction to the apparent latitude depends on the apparent longitude. In applying this correction it is only necessary to employ the first two figures of the value of the natural cosine. In the present instance, therefore, +·46 × ·59 is the correction to be applied. Hence the true latitude of the spot is −16°·5 S. We stated that L for the date and time of the drawing was 266°·56. This value has been thus derived: At noon on August 9 L for the central meridian was 265°·25, but the outline of the spot was traced at $9^h\ 37^m$, or $2^h\ 23^m$ before noon; and although the difference in longitude of the central meridian for an interval of twenty-four hours varies with the time of the year, yet it is sufficiently accurate to reckon the mean difference in longitude for this period as 13°·22. Hence the differences for hours and minutes can easily be deduced. For $2^h\ 23^m$ the correction to be applied is +1°·31. Hence L at the time of the drawing was 266°·56, and the true longitude of the spot becomes 266°·56 −53°·5=213°·1. The Greenwich results for the same spot are lat. −16°·1, long. 213°·1. As the projections are made for every whole degree, the error, either in the parallels or the meridians of any projection used, ought not to exceed what would correspond to a difference of half a degree of polar tilt, since each whole degree projection is used only for dates corresponding to the nearest half degree at either side of it. This error is reduced by the use of the table of natural cosines as explained. The error of the meridians due to a difference of half a degree of tilt is in the spot zones about two-tenths of a degree as a maximum. This error also can be eliminated by measuring the longitude a second time by means of a projection corresponding to half a degree polar tilt in the reverse sense and taking the mean of the two readings. In this manner the error in longitude can be reduced to about 0°·1.

As pointed out in the "Researches on Solar Physics," by W. De la Rue, B. Stewart, and B. Lœwy (*Phil. Trans.* 1869, p. 10), the measured distance of a spot from the centre of a drawing is greater than its true angular distance from the central meridian of the Sun. In using orthographic projections for deducing the positions of the sun-spots an error due to this difference will always enter into the measurements. But if a spot is followed across the disc the errors on both sides of the central meridian, being of opposite sign, will neutralise each other in determining the mean longitude of the spot. But this is not the case for the latitudes, which ought therefore to be always greater than the true latitude. The maximum value of this error is between 0°·2 and 0°·3 for projections of 10½ inches diameter. Hence in using projections such as those described the longitudes should be much nearer the Greenwich standard positions than the latitudes, and for the latter the maximum cumulative error from all causes ought not to exceed 0°·4. Having at hand 82 positions of sun-spots for the year 1889, originally measured for a different object, we have compared them with the Greenwich positions for the same spots on the same dates. Of these measures 5 of the longitude and 9 of the latitude have been rejected as anomalous, either because the spots were very near the limb where measures are made with difficulty, or because they were changing their positions rapidly. Of the remaining 77 positions in longitude 3 differ 0°·6, 6 differ 0°·5, and 5 differ 0°·4 from the Greenwich positions, the remaining 63 varying between the limits 0° to 0°·3. Of the 73 positions in latitude used in the comparison 5 differ 0°·7, 5 differ 0°·6, 4 differ 0°·5, and 4 differ 0°·4 from the Greenwich positions, while 55 are within the 0°·3 limit. If, however, we take the mean positions of the groups the accordance is much closer, as the following table shows. For the omitted groups from the table either we have no positions from the drawings, or in two cases, groups 2099 and 2100, the results are vitiated by the anomalous positions alluded to above.

No. of Group.	Greenwich Mean. Long.	Lat.	Stonyhurst Mean. Long.	Lat.	Difference G—S. Long.	Lat.	No. of Comp.
	°	°	°	°	°	°	
2083 (*a*)	318·64	− 7·89	318·54	− 8·11	+0·10	−0·22	8
2085	311·53	+ 5·88	311·53	+ 5·75	0	+0·13	4
2086	315·10	+ 5·10	315·20	+ 4·72	−0·10	+0·38	5
2088	217·43	− 1·55	217·43	− 1·83	0	−0·28	4
2090 (*a*)	31·10	− 5·42	31·08	− 5·60	+0·02	−0·18	11
2092	36·31	− 7·91	36·30	− 8·32	+0·01	−0·41	9
2094 (*a*)	343·55	−23·40	343·65	−23·45	−0·10	−0·05	2
2097 (*a*)	163·78	−20·35	163·88	−20·58	−0·10	−0·23	6
2097 (*b*)	155·43	−21·60	155·53	−21·65	−0·10	−0·05	4
2103	151·85	−20·65	151·73	−21·12	+0·12	−0·47	6
2110	101·13	−26·53	101·03	−26·63	+0·10	−0·10	3

It is to be understood, of course, that the drawings can also be measured and reduced according to the methods detailed in Carington's researches on solar physics, our present object being to show how closely the simple method of projection measurement approximates to those that are more accurate. The appended verification of the exactness with which the projections have been drawn, and a method of measurement deduced from them which is independent of their use, is due to the Rev. James Cullen, of this college.

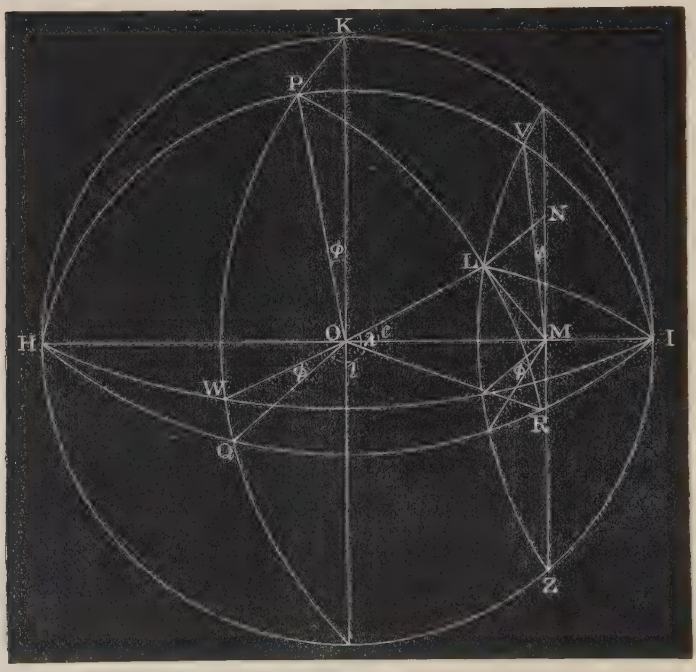


FIG. 1.

1. The discs have been verified in the following manner :— In fig. 1 let P be the pole of the Sun, HQRI his equator, HWI the GC of which K is the pole. If, then, L is a spot, its co-ordinates (heliographic) are QR and RL. It is clear that as P moves in the meridian KPWQ, L will move in the small circle VLZ, the radius of which is $LM=OL \sin c = r \sin c$ (r=radius of sphere). But c=LI, the hypotenuse of the right-angled triangle LIR ; LR $=\lambda$ (the latitude) ; $RI=90-l$ (l being the longitude).

$$\therefore \cos c = \cos \lambda \sin l.$$

Hence

$$LM = r\,(1-\cos^2 \lambda \sin^2 l)^{\frac{1}{2}},$$

and if $\angle \mathrm{LIV} = \theta$

$$\mathrm{MN} = \mathrm{LM} \cos(\theta + \phi).$$

Now $\theta = \mathrm{I}$, the angle of the triangle PLI; $\mathrm{PL} = 90° - \lambda$; $\mathrm{PI} = 90°$; and

$$\mathrm{LI} = c = \cos^{-1}\{\cos \lambda \sin l\};$$

whence applying the formula

$$\cos \mathrm{A} = (\cos a - \cos b \cos c)/\sin b \sin c,$$

where $\mathrm{A} = \theta$, $a = \mathrm{PL}$, $b = 90°$, we get

$$\cos \theta = \sin \lambda/(1 - \cos^2 \lambda \sin^2 l)^{\frac{1}{2}}.$$

Therefore substituting for θ in $\mathrm{MN} = \mathrm{LM} \cos(\theta + \phi)$, we have

$$\mathrm{MN} = y = r(\cos \phi \sin l - \sin \phi \cos \lambda \cos l) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\mathrm{OM} = x = r \cos c = r \cos \lambda \sin l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The curves giving the meridians and parallels of latitude are found by making $l = a$ (a constant) and eliminating λ between (1) and (2) or making $\lambda = \beta$ and eliminating l.

Knowing ϕ, λ, and l, we can find the ordinates and abscissæ of any of the points of intersection of the parallels of latitude and meridians of longitude on the discs. This has been applied to the Stonyhurst projections, and they were found to be absolutely correct.

2. As stated in the paper, a proportional table was constructed empirically from measurements taken on the discs in order to apply a correction to the latitude due to the apparent longitude of the spot. That this is a table of natural cosines can be thus shown.

For a projection ϕ_1, say, we have from (1)

$$y_1 = r(\cos \phi_1 \sin \lambda - \sin \phi_1 \cos \lambda \cos l),$$

and similarly for ϕ_2 another projection

$$y_2 = r(\cos \phi_2 \sin \lambda - \sin \phi_2 \cos \lambda \cos l)$$

$$\therefore y_1 - y_2 = r\{(\cos \phi_1 - \cos \phi_2) \sin \lambda - (\sin \phi_1 - \sin \phi_2) \cos \lambda \cos l\}$$

and as ϕ_1 and ϕ_2 always lie between 0° and 8°, $\cos \phi_1$ and $\cos \phi_2$ lie between 1 and ·990; so that $\cos \phi_1 - \cos \phi_2$ never can be greater than ·01, and as two successive projections are generally used, it really becomes $\cos \phi° - \cos(\phi + 1)° = ·001$ (nearly); and as this is multiplied by $\sin \lambda$ (λ being small for spots), we may neglect the first term in the formula. Hence we have

$$y_1 - y_2 = r(\sin \phi_2 - \sin \phi_1) \cos \lambda \cos l,$$

which varies as $\cos l$ as in the case of equators, for when $\lambda = 0$

$$y_1 - y_2 = r(\sin \phi_2 - \sin \phi_1) \cos l$$

by the general formulæ.

Between 0° and 64° on a disc whose radius is 133·35 mm. (or $5\frac{1}{4}$ in.) an error of 0·1 mm. in the equator cannot cause a greater error than 6′ or 0°·1 in the longitude. But at the limb *one degree* is represented by 0·02 mm.

3. From the consideration of the fundamental equations (1) and (2) a method of measurement of the position of sun-spots can be derived directly from the drawings without the use of any discs. From the *Companion to the Observatory* the position-angle of the north pole of the Sun, &c., can be derived for any day. Let NWSE (fig. 2) represent the picture, draw the polar

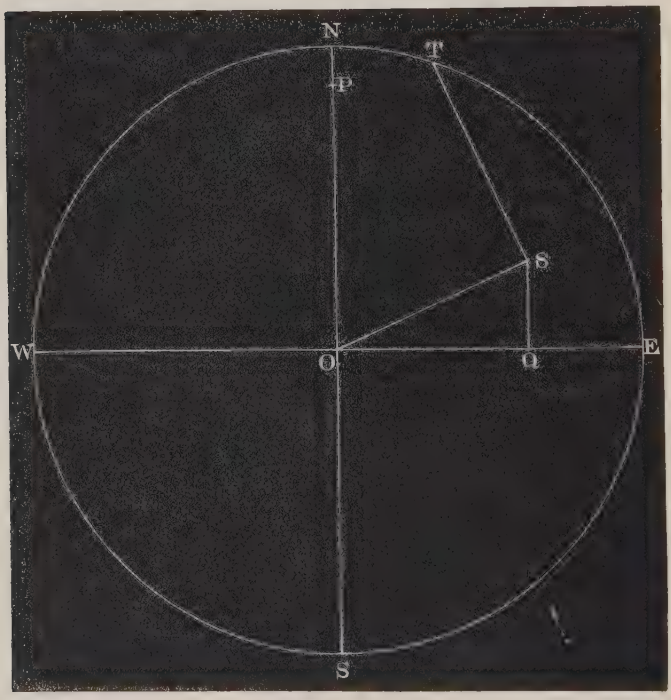


Fig. 2.

and equatorial diameters NS and WE. Then from (1) and (2) we get

$$\cos l = (r^2 \cos^2 \lambda - x^2)^{\frac{1}{2}} / r \cos \lambda,$$

and

$$(y - r \cos \phi \sin \lambda)^2 = \sin^2 \phi \, (r^2 \cos^2 \lambda - x^2)$$

$$\therefore y^2 - 2ry \cos \phi \sin \lambda + r^2 \cos^2 \phi \sin^2 \lambda - r^2 \sin^2 \phi \cos^2 \lambda + x^2 \sin^2 \phi = 0;$$

i.e.

$$r^2 \sin^2 \lambda - 2ry \cos \phi \sin \lambda + x^2 \sin^2 \phi - r^2 \sin^2 \phi^2 + y^2 = 0,$$

or

$$(r \sin \lambda - y \cos \phi)^2 = \sin^2 \phi \{r^2 - (x^2 + y^2)\}$$

$$\therefore \sin \lambda = [y \cos \phi + \sin \phi \{r^2 - (x^2 + y^2)\}^{\frac{1}{2}}]/r.$$

The positive sign must be taken in extracting the square root, since from (1)

$$\sin \lambda > y \cos \phi / r$$

or

$$> \cos^2 \phi \sin \lambda - \sin \phi \cos \phi \cos \lambda \cos l$$

if λ, ϕ, and l are positive.

In practice the following method will facilitate the evaluation of λ. Let S be the spot; join O and S, draw S T perpendicular to O S; join O and T, and draw S Q perpendicular to O E. Then $OQ = x$ and $SQ = y$; $OT = r$. We have therefore

$$ST^2 = OT^2 - OS^2 = r^2 - (x^2 + y^2).$$

Since

$$OS^2 = x^2 + y^2 \therefore ST = \{r^2 - (x^2 + y^2)\}^{\frac{1}{2}}$$

Hence

$$\sin \lambda = (y \cos \phi + ST \sin \phi)/r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\sin l = x/r \cos \lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

λ being found from (3).

We may write

$$\sin \lambda = y \, . \, \alpha + ST \, . \, \beta,$$

where $\alpha = \cos \phi / r$, and $\beta = \sin \phi / r$, these quantities being determined each day: y is negative if measured southwards, β is negative when ϕ is negative, and λ is S. lat. when $\sin \lambda$ is negative.

A convenient way of making use of this method would be to have a right-angled elbow-ruler graduated along both edges in inches or millimetres. Let the zero point be at the elbow; the reading to the centre of the spot gives the x and the reading at right angles to the centre of the disc, the y. Now place the zero point at the centre of the spot, with one edge passing through the centre of the picture; the reading where the other edge cuts the circle gives us ST, and hence the heliographic coordinates can be easily calculated from equations (3) and (4).

4. In the cases where the correction for refraction is needed a modification of the formulæ given in the Introduction to the Greenwich Spectroscopic and Photographic Results can be employed.

Mercury. By Percival Lowell.

(*Communicated by the Secretaries.*)

The markings on the planet's disc proved at once to be conspicuous. To reduce them to a scale I may say that they were easier to see than all the markings on *Mars* except the most salient ones, such as the so-called Seas. So visible were they at all times—for the air had to be distinctly bad to obliterate them—that observations had not been made more than a day or two, before the rotation period of the planet was patent.

Further communication upon this planet I intend to give later.

Lowell Observatory:
1896 *October* 21.

[In addition to those reproduced, Mr. Lowell sends several prints of drawings of *Mercury*, which have been placed in the Library.—Eds. *M.N.*]

Determination of the Rotation Period and Surface Character of the Planet Venus. By Percival Lowell.

(*Communicated by the Secretaries.*)

The rotation period of *Venus* has hitherto been a matter of doubt. I beg therefore to communicate to the Society the following observations made by me during August, September, and October of this year, which prove that rotation to be performed in the same time as the planet's orbital revolution. I send drawings by me and by my assistant, Mr. Drew. Comparison of the drawings will show their conclusive character, and from them I have been able to construct the accompanying chart.

In the construction of the chart I have taken for the zero longitude that longitude which is upon the central meridian as observed from the Sun when the planet is in the line of apsides of its orbit, either at perihelion or at aphelion, such being the most natural origin to adopt.

In orienting the different drawings of the disc the libration in longitude due to the excentricity of the planet's orbit is added to or subtracted from the value resulting from the planet's orbital revolution. In the case of *Venus* the resulting change is very small, never amounting to more than $47'$ at its maximum.

The planet's axis of rotation is substantially perpendicular to the plane of its orbit, as the observations have not yet disclosed any deviation from this position.

In the names for the markings I have selected such as seemed appropriate to the Planet of Love. The markings themselves are

MERCURY, 1896.

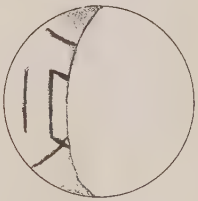
Sep. 14, 2h 52m
λ = 98°

Sep. 19, 5h
λ = 115°

Sep. 23, 3h 12m
λ = 130°

Sep. 23, 3h 30m
λ = 130°

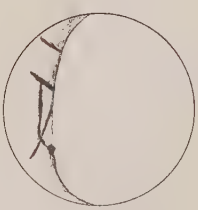
Sep. 23, 4h
λ = 130°



Sep. 23, 4h 42m
λ = 130°

Sep. 25, 2h 13m
λ = 138°

Sep. 26, 3h 22m
λ = 141°



Sep. 26, 3h 56m
λ = 141°

Percival Lowell, del.

VENUS, 1896.

Oct. 13, $2^h 55^m$ to $3^h 4^m$
$\lambda = 37°$

Oct. 14, $1^h 59^m$ to $2^h 12^m$
$\lambda = 37°$

Oct. 14 $3^h 13^m$ to 18^m
$\lambda = 37°$

Oct. 14, $3^h 50^m$
$\lambda = 37°$

Oct. 15, $0^h 5^m$ to 14^m
$\lambda = 37°$

Oct. 15, 1^h
$\lambda = 37°$

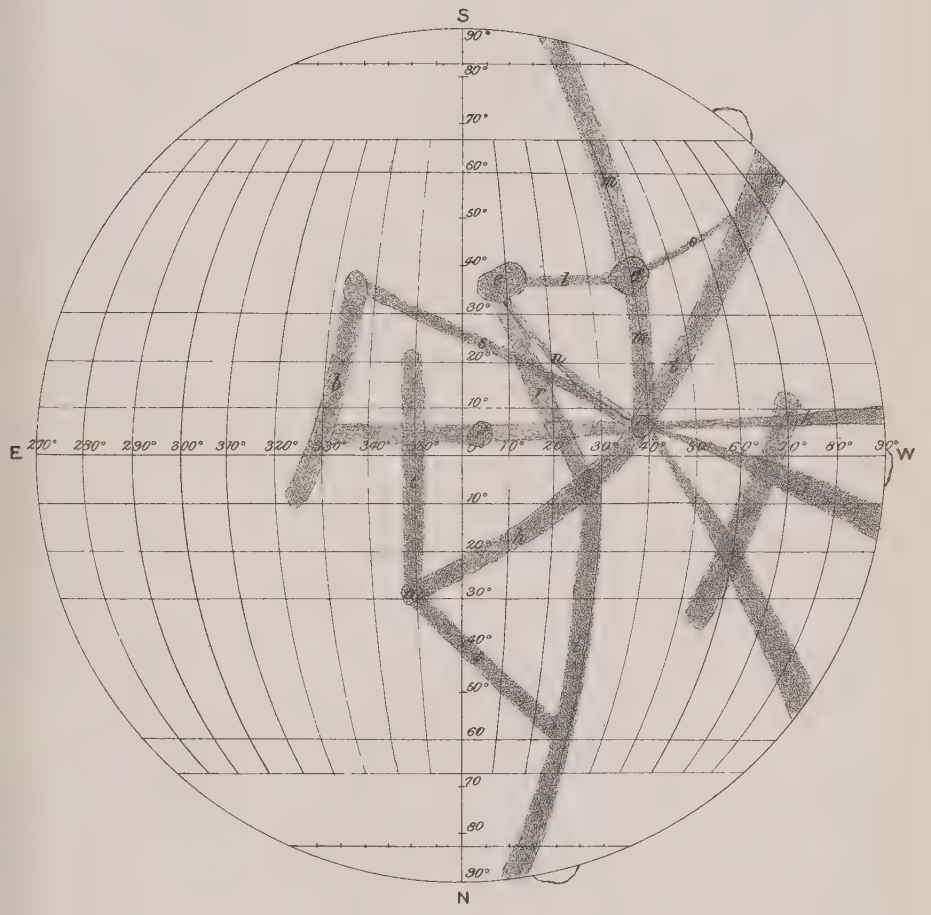
Oct. 15, $2^h 10^m$ to 21^m
$\lambda = 37°$

Oct. 15, $3^h 48^m$ to 54^m
$\lambda = 37°$

Oct. 15, $4^h 37^m$
$\lambda = 37°$

Percival Lowell, del.

CHART OF VENUS.



Percival Lowell, del.

long and narrow; but, unlike the finer markings on *Mars*, they have the appearance of being natural, not artificial. They are not only permanent, but permanently visible whenever our own atmospheric conditions are not so poor as to obliterate all detail on the disc. They are thus evidently not cloud-hidden at any time. But the whole disc, dark and light portions alike, is brightened as by a luminous atmosphere. When compared with the appearance of the disc of *Mercury* or of our own Moon, the appearance of the disc of *Venus* is such as to make the presence of a very substantial atmosphere evident. Measures of the polar and equatorial diameters confirm this deduction by disclosing by comparison with *Mercury* a visible twilight arc.

There is no distinctive colour in any part of the planet other than its general brilliant straw-colour hue. The markings, which are of a straw-coloured grey, bear the look of being ground or rock, and it is presumable from this that we see simply barren rock or sand weathered by æons of exposure to the Sun. The markings are perfectly distinct and unmistakable, and conclusive as to the planet's period of rotation.

There is no certain evidence of any polar caps. Only once have I seen anything which might be so construed, and even that is perfectly explicable by a greater luminosity of the surface at those points. The planet's surface presents as dead an appearance as does that of our own Moon.

Lowell Observatory:
1896 *October* 21.

[In addition to those reproduced, Mr. Lowell sends several prints of drawings of *Venus*, which have been placed in the Library.—Eds. *M.N.*]

Index to Map of Venus.

a Eros
b Psyche regio
c Hermione regio
d Ashtoreth regio
e Ashera
f Anchises regio
g Hero regio
h Aphrodite regio
i Æneas regio
j Anteros regio
k Adonis regio
l Dione regio
m Paris regio
n Hymenæus regio
o Hephæstos regio
p Istar
q Somnus regio
r Cytherea regio
s Cyprus regio
t Pothos
u Bilit
v Astarte regio
w Libentina regio

Lowell Observatory:
1896 *October.*

Determination of the Diameter and the Compression of the Planet Mars from Observations with the Repsold Heliometer of the Royal Observatory, Göttingen. By Professor W. Schur.

(*Communicated by the Astronomer Royal.*)

The question of the compression of *Mars* is still open, as the former measures of the equatorial and the polar diameter may be influenced by errors of estimation to which the eye is liable in the measurement of discs in different directions with respect to the vertical line.

But since small reversion prisms are used on the eyepiece of the heliometer, with which the image of a celestial body, *e.g.* the line joining a double star or the diameter of a planetary disc, can be made to appear in any direction with respect to the vertical line, it is possible to get free from these subjective influences.

I have, therefore, awaited the opposition of *Mars* of this year, which is so favourable on account of the high declination of the planet, with great anxiety to observe the equatorial and the polar diameter with the said precautions.

All instrumental errors and the very small correction for defect of illumination according to the ephemeris of Mr. Marth are taken into account.

The diameter of the object-glass is 162 millimetres and the power used 174.

The following are results of four nights, where v and h denote the measurements with apparent vertical and horizontal movements of the images in the field of the eyepiece.

Date.	Mean Time Göttingen.	Areographic latitude.	Measured diameter.	Diameter in mean distance from the Sun.	Mean of v and h.
1896.	h m	°	"	"	"
Dec. 2	11 45	90	16·81v	6·19	6·15
			16·60h	6·11	
		0	17·06v	6·28	6·30
			17·16h	6·32	
		0	17·03h	6·27	6·23
			16·79v	6·18	
		90	16·66h	6·13	6·10
			16·49v	6·07	
11	10 19	90	16·52h	6·14	6·13
			16·45v	6·11	
		0	17·04h	6·33	6·30
			16·85v	6·26	
		0	16·92v	6·29	6·32
			17 09h	6·35	

Date.	Mean Time Göttingen.	Areographic latitude.	Measured diameter.	Diameter in mean distance from the Sun.	Mean of v and h.
1896.	h m	°	″	″	″
Dec. 11	10 19	90	16·55v	6·15	6·14
			16·47h	6·12	
16	8 2	90	16·18h	6·12	6·10
			16·06v	6·07	
		0	16·44h	6·22	6·11
			16·43v	6·21	
		0	16·44v	6·22	6·21
			16·40h	6·20	
		90	16·11v	6·09	6·09
			16·11h	6·09	
17	9 36	0	16·46h	6·26	6·24
			16·34v	6·21	
		90	16·24h	6·17	6·15
			16·10v	6·12	
		90	16·06v	6·10	6·10
			16·06h	6·10	
		0	16·25v	6·18	6·24
			16·54h	6·29	

December 2 the images, principally in the beginning, were less steady, but on the other days pretty and sharp.

Taking the mean of the numbers in the last column, one gets for the axes of the Martian disc in mean distance from the Sun (log=0·18289).

1896.				
Dec. 2	$2a = 6''{\cdot}265$	$2b = 6''{\cdot}125$	$a = \frac{a-b}{a} = \frac{1}{44{\cdot}7}$	
11	6·310	6·135	$\frac{1}{36{\cdot}1}$	
16	6·210	6·095	$\frac{1}{54{\cdot}0}$	
17	6·240	6·125	$\frac{1}{54{\cdot}3}$	

The mean for the four observing nights gives the compression nearly $=\frac{1}{47}$, and the diameters are—

$$2a = 6''{\cdot}256 \qquad 2b = 6''{\cdot}120.$$

Reduced to the distance, Earth-Sun the diameters are—

$$2A = 9''{\cdot}53 \qquad 2B = 9''{\cdot}32.$$

From the results of these observations, which in my opinion are perfectly free from any disturbing influence, it seems to be not doubtful that the compression of the planetary disc is nearly $\frac{1}{47}$.

Remarks on the Physical Observations of Jupiter, made at the Adelaide Observatory, 1884–93. By E. P. Sells.

(*Communicated by Sir Charles Todd.*)

Drawings of *Jupiter* were made at the Adelaide Observatory, with the 8-inch Cooke refractor, from 1884 till 1893, with the exception of 1890.

It was early recognised that the utility of the drawings would be much increased if the two following conditions were complied with:—

(1) The drawings to be arranged for publication, not in the usual way of separate discs for each drawing, but on some cylindrical form of projection, allowing of easy comparison between the results obtained on different nights.

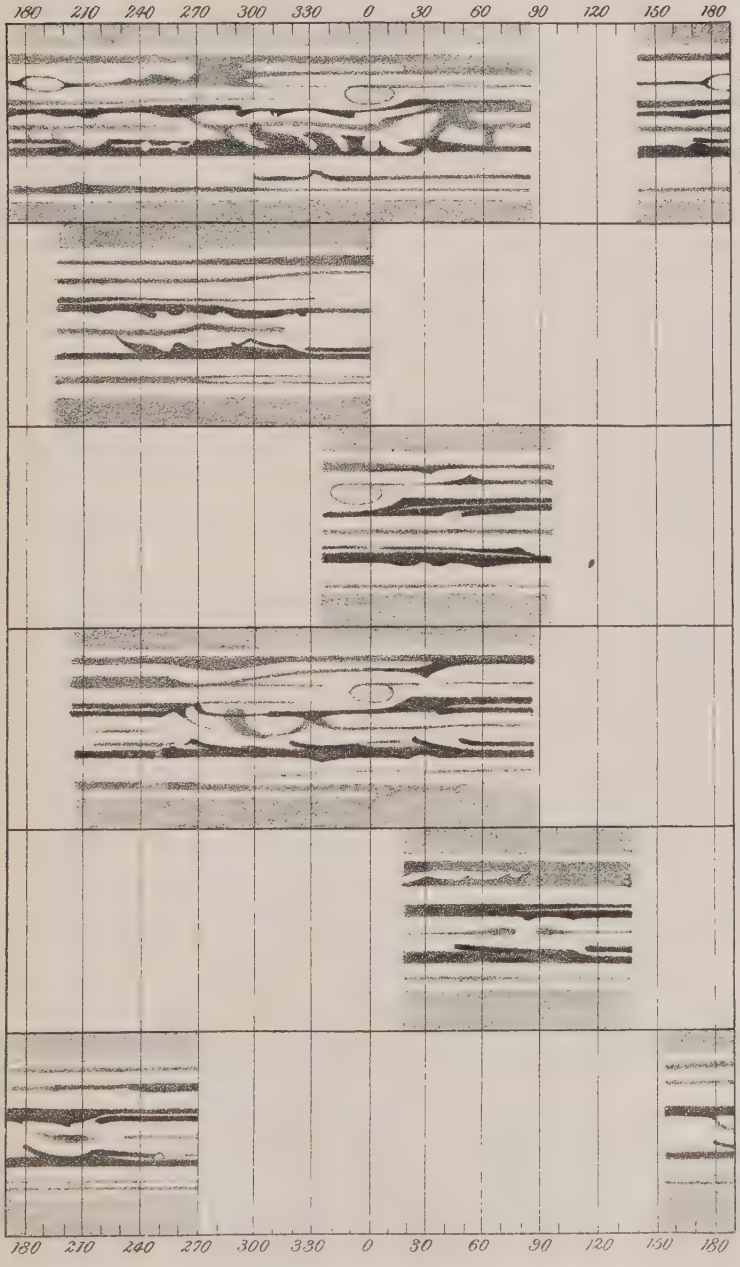
(2) As many drawings as possible to be taken on each night of observation, four or five drawings on one night being preferable to one drawing on each of four or five nights.

With regard to condition (1), the form of projection chosen was the orthographic, being the most natural projection for *Jupiter*. The drawings at the telescope were made on blank elliptical discs, as is usual, and were afterwards plotted on a chart having its polar diameter equal to that of the elliptical disc, the longitudes being taken from Marth's "Ephemeris," System II.

It will be seen from the accompanying planisphere that each night of observation has a chart to itself, and the different charts are arranged one below the other, and are separated from each other by only a thin black line, thus allowing of easy comparison *inter se*. The number of charts on a sheet is of necessity limited by the size of the publication of which they form a part, but it is, of course, an advantage to have as many charts as possible on each sheet. No latitude parallels are shown, as they would only confuse the eye, but meridians are given to every 30°, and along the top and bottom of the sheet to every 10°.

With regard to condition (2), drawings were made as nearly as possible at intervals of fifty minutes, answering to a difference of 30°·2 Jovian longitude. Attempts were made to secure sets of drawings showing the whole of *Jupiter's* surface during one revolution of the planet, but in no case was this satisfactorily accomplished, although on 1889 June 23–24 G.M.T. nine drawings were secured, of which the first and last slightly overlapped, so that on the planisphere the whole disc is delineated. On all occasions, however, observations were interrupted either by cloud or by a south-easterly breeze, the lightest breath of air from that quarter sufficing to destroy the quality of the definition, so that the true forms of the markings could not be made out, and all delicate detail vanished. Great attention has been paid at Adelaide to giving the markings their true forms (as they

JUPITER, 1889.



E. P. Sells. del.

appeared to the observer) and definite, but not necessarily hard, outlines have been drawn whenever possible.

In all, 217 drawings have been plotted on the charts, ranging in date from 1884 February 25 to 1893 December 11.

At no time have the Polar and Temperate Regions shown much detail, but this is, of course, partly due to the foreshortened view that is obtained of those portions of the planet. The outburst of small dark spots in 1891 on the southern edge of the North Temperate Belt was remarkable, especially when taken in conjunction with the somewhat similar outburst in 1880 and 1881, but this has been sufficiently commented on by other observers. After the outburst had subsided, and during the apparitions of the planet in 1892 and 1893, this belt had become very dark, and towards the end of 1893 was the darkest marking on the planet. In South Temperate Regions perhaps the chief feature has been the trains of small round white spots that sometimes make their appearance, of which there are numerous instances among our drawings. Also noticeable are the short dark belts that form in about latitude 25° S., of which the most conspicuous examples are the two that were visible in 1891, one of them being of a very dark reddish colour. An interesting feature was the appearance of a small dusky spot in 1884, 1886, and 1889, interrupting the line of the South Temperate Belt. The spots may not have been identically the same, but were very similar in appearance. Two drawings, showing the spot as it appeared in 1884 and 1889, respectively, accompany this paper. In 1884 it was seen on March 20 (long. 144°·5) and April 6 (long. 128°·9), and then appeared as a small circular dusky spot with a clear space all round it, interrupting the line of the narrow Belt 6 (see *Monthly Notices* for April 1891, p. 404), while the south polar shading, separated from Belt 6 by a very narrow white space, had its northern contour hollowed out immediately south of the spot, and also had a darker degree of shading at the same place.

In 1886 a pear-shaped spot was seen in the same latitude. Again it interrupted the line of Belt 6, and again the south polar shading had a darker shade immediately south of the spot; but there was no dip southwards of the edge of the shading as in 1884, perhaps because the white space was much broader between the shading and Belt 6. The spot was noticed on February 3 (long. 73°), February 8, and February 22, but after that was not seen again until 1889, when a similar spot made its appearance in the same latitude. In 1889 the spot and Belt 6 were darker and broader than formerly, and the belt appeared joined on to a band of shading to the south. The longitude of the spot 1889 June 25 at $0^h\ 24^m$ G.M.T. was 141°·2.

The equatorial regions during some apparitions of the planet showed a large amount of detail, but during others the markings were few. There have always been, during the period 1884–1893 the two chief belts, the North and South Equatorial Belts, though in 1884 the North Equatorial Belt was broken up along

part of its length into short disconnected belts. In 1885 the belt was continuous and had become far more conspicuous, while in 1886 it showed a fair amount of structural detail. Only a few drawings were taken here in 1887 and 1888, but the belt still showed a quantity of detail, as was also the case in 1889. No observations were taken in 1890, but in 1891 the belt had a more regular outline, and had become very narrow and dark along part of its course. It appeared to consist of two different layers of matter, one very dark and narrow, and the other fainter and broader, the dark, narrow layer coming to a clearly defined end within the lighter broad layer.

The two dark spots on the northern component of the double portion of the belt, in longs. 220°–255°, were seen and drawn on five nights.

In 1892 and 1893 the belt had become still narrower, and showed very little detail, but towards the end of 1893 it was getting broader again, and was double for part of its length.

The South Equatorial Belt has always been a conspicuous marking. It is generally, but not always, divided longitudinally by a white streak, while sometimes a white streak will cross it at a considerable angle. Perhaps the most usual appearance of this belt, as seen at Adelaide, has been that of a comparatively lightly shaded belt, divided longitudinally by a white streak, and having superposed upon its northern component other dark belts, short in length, and frequently with a club-shaped termination projecting into the bright equatorial zone, and forming the bases from which spring the festoons and other shadings that appear on that zone.

When observations were begun in 1884 the Great Bay in the southern edge of the South Equatorial Belt, north of the Great Red Spot, was a conspicuous feature, and was symmetrically shaped, the angular shoulders on either side being similar to one another. On 1884 April 20 at $20^h\ 37^m$ G.M.T. the *preceding* angle of the Great Bay was found to be exceptionally sharp and well defined, and appeared of a deep blood-red colour, fading off towards the preceding limb. The colour was most marked. In 1885 this angular shoulder on the *preceding* side of the Great Bay had disappeared, except that a narrow faint line was sometimes visible in the position of its southern edge, nor did it appear again up to the end of 1893.

In 1892 and 1893 the belt showed very little detail, and there was a striking absence of markings on the bright equatorial zone, all the equatorial regions of the planet appearing to be in a state of comparative quiescence.

[In addition to the chart reproduced, Mr. Sells also sent a chart for 1888, which is placed in the Library.]

Note on the Magnitude of η Argûs, 1896. By R. T. A. Innes.

(*Communicated by Dr. Gill, H.M. Astronomer.*)

Four determinations of the brightness of this star were made here in 1896, viz.—

	Mag.
Apr. 26	7·6
May 17	7·5
June 28	7·6 orange yellow
July 13	7·6
Means 1896·43	7·58

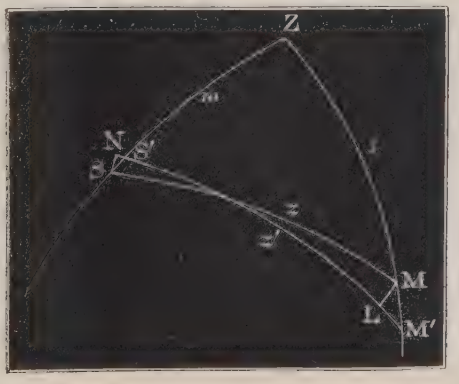
The telescope used was the 7-inch equatorial, and the comparison stars and magnitudes are those given for that purpose in the *Uranometria Argentina*.

The magnitude of *η Argûs* was also determined here in 1886 March (see *M.N. R.A.S.* vol. xlvi. page 340) by Mr. W. H. Finlay, on which occasion it was found to be 7·6 mag., the same comparison stars being used.

Royal Observatory, Cape of Good Hope:
1896 *December* 16.

A Method of Clearing a Lunar Distance. By F. C. Penrose.

Various methods have been proposed for facilitating the operation in nautical astronomy of clearing the lunar distance. One was brought before the Society in 1884 by Mr. John Merrifield, which is printed in the *Monthly Notices* for April of that year. The plan I here venture to propose has thus much in common with that above mentioned, viz. that it deals chiefly with the small triangles which result from parallax and refraction, but it is dissimilar in all other respects.



The advantage looked for is that, whilst all desirable accuracy is obtained, it can be worked with logarithms restricted to four places, and thereby lightens the labour of calculation, and to some degree also the chance of errors in computation.

Let Z be the zenith, M′ and S′ the apparent, and M and S the true places of the Moon and Sun or star, and z' and z the apparent and true distance respectively, and let the sides of the small triangles about M and S be named—MM′, δs, M′L δz and SS′ and SN δm and $\delta z_{,,}$ respectively. It can easily be shown that when these triangles are very small *

$$\delta z = \delta s \left(\frac{\cos m \,.\, \sin s - \sin m \,.\, \cos s \,.\, \cos Z}{\sin z} \right)$$

and the same, *mutatis mutandis*, applies to $\delta z_{,,}$. The triangle connected with S will almost always be small enough, but that at M, when great accuracy is required, will generally need a small correction, but one that is very easily applied.

Taking an example from *Raper's Navigation* (p. 289 of 9th edit.)—

Apparent alt. of ⊙ 47° 31′, of ☾ 36° 52′, app. dist. 48° 20′ 29″. Sun's correction 47″, that of ☾ 45′ 35″.

First calculate the angle Z—in this step it is best to use logarithms to five figures—and whilst taking out the logarithms necessary for this step, take out also—to four places—those of the sin and cos of the side s diminished by half the parallax in altitude, and of m increased by $\frac{1}{2}$ 47″.

m' 42° 29′ 0″	9·82954	Also 9·8296 and 9·8677 log cos m
s' 53 8 0	9·90311	„ 9·9009 „ 9·7819 „ s
z' 48 20 29	9·73265	
2)143 57 29		
71 58 44	9·97815	
−48 20 29		
23 38 15	9·60309	
	9·58124	Z = 65° 42′ 45″. Its log cos 9·6142
	−9·73265	
	9·84859	

* Cos z = cos m . cos s + sin m . sin s . cos Z.

In this inquiry Z is always constant, and whilst considering the effects upon s and m separately, the opposed side m or s will be constant.

$$\therefore -\sin z \frac{dz}{ds} = -\cos m \,.\, \sin s + \sin m \,.\, \cos s \,.\, \cos Z,$$

$$\text{and } \therefore dz = ds \left(\frac{\cos m \,.\, \sin s - \sin m \,.\, \cos s \,.\, \cos Z}{\sin z} \right).$$

Then using the formula above given

L cos m 9·8677	L sin m 9·8296		
L sin s 9·9009	L cos s 9·7819		
+9·7686	9·6115		
	L cos Z 9·6142		
No. ·5869 +	9·2257	No. ·1682 negative	

Then

·5869		
−·1682		
·4187	Its log 9·6219	
	−L sin z 9·8734	
	9·7485	
	+ log sin δs 8·1225	
	7·8710	Log sin 25′ 32″·6. First approx. of δz.

For $dz_{//}$ use the same elements, but transposed :

Log cos s . sin m 9·6115	Log sin s . cos m 9·7686		
	L cos Z 9·6142		
No. ·4088	9·3828	No. ·2414 negative	
−·2414			
·1674	Its log 9·2238		
	−L sin z 9·8734		
	9·3504		
	+ Log sin 47″ 6·3577		
	5·7081	Log sin 0′ 10″·5 = $\delta z''$	

Hence the first correction of the distance is

$$25' \ 32''{\cdot}6 - 10''{\cdot}5 = 25' \ 22''{\cdot}1,$$

and the distance becomes 47° 55′ 7″ as a first approximation. Now reduce the original distance by half the calculated correction, and use the sin of this value in the previous computation of δz ; or, what amounts to the same thing, apply to the final term the difference due to this reduced value of log sin z, which in this case (9·8719 instead of 9·8734) amounts to +·0015, and the final term to 7·8725, making $dz_{/}$ = 25′ 38″.

The value of $\delta z_{//}$ will not be altered appreciably, so the final correction is 25′ 27″·5, and the distance 47° 55′ 1″·5.

A complete solution of this example, using logarithms to seven places, gives the distance 47° 55′ 0″·88.

It may be seen by the formula that when the angle Z is less than 90° if the second and negative term is greater than the first, the distance will be increased by the Moon's parallax in altitude, but in extreme cases the solar (or stellar) refraction may modify the general rule.

Notes on Meteors observed at Penarth, Glamorgan, on 1896, *Nov.* 14. By George Carslake Thompson, LL.M., and H. W. Lloyd Tanner, M.A., F.R.A.S.

Our attention was mainly directed to the region of *Leo Major*. A building behind us cut off the western sky, some trees cut off the south, and we faced east or north-east. We both watched for about four hours after midnight with an interval specified in the following table. After 4 A.M. the observations were continued till 5.20 A.M. by Mr. Carslake Thompson alone. Each observation was recorded and rough notes were written at the time by G. C. T.

The entries in column A1 refer to meteors from the Sickle in *Leo*; A2, from the region above the Sickle; A3 from the region of β *Leonis* or the region between β and the Sickle. The entries in column B are of meteors from other directions. The directions of the meteors noted in column C were not determined.

Time.	A 1	A 2	A 3	B	C	Notes.
Soon after midnight	...	...	...	1	...	
12 35	1	...	...	...	...	
12 45	...	...	1	...	...	
12 52	1	...	...	...	...	
12 57	1	...	...	...	...	Short course with fine train; appeared 3 or 4 degrees eastward of Sickle; direction of course, perpendicular to and bisecting the line joining γ and ζ Leonis.
1 11	...	1	...	...	...	
1 30	1	...	...	...	...	Very faint and very short course; appeared very near ζ Leonis; course inclined upwards at perhaps 30° to line joining γ and ζ.
1 34	1	...	...	...	...	
1 37	...	...	...	...	1	
1 47	...	...	1	...	...	
1 54	1	...	...	...	...	Fine train from direction ζ Leonis Maj. towards η or ζ Ursæ Maj.
1 54½	1	...	...	1	...	In same line; might have been same one coming into the air again.
1 58	...	...	...	...	...	
2 1	1	...	...	...	...	A very fine one; passed very close to ψ and γ Ursæ Maj. (perhaps half a degree above them) in a line parallel to the line joining these stars. Train persisted 2 or 3 seconds.

Time.	A 1	A 2	A 3	B	C	Notes.
2 6–7	...	1	...	...	...	
2 6–7	...	...	...	...	1	
2 6–7	...	...	...	...	1	
2 14	1	...	...	...	...	
2 20	...	1	...	...	...	A rather faint one; downwards, past, and to ζ and γ Leonis Maj. (counted as from general direction of Leo, but query; compare two at 4·25 not so counted).
2 26	...	...	...	1	...	
2 27	1	...	...	...	...	
2 29	1	...	...	...	...	
2 30	1	...	...	...	...	
2 30½	1	...	...	...	...	
2 36	1	...	...	...	...	Persistent train.
2 36–7	...	...	...	...	1	
2 36–7	...	...	...	...	1	
2 41	...	...	...	1	...	
2 45	...	1	...	...	...	
2 53	1	...	...	...	...	
2 58	...	1	...	...	...	
3 0	1	...	...	...	...	
3 1	...	...	1	...	...	
3 4	...	1	...	...	...	
3 5	1	...	...	...	...	
3 5½	1	...	...	...	...	
3 6	...	...	...	1	...	
3 6	1	...	...	...	...	
3 7	...	...	...	1	...	
3 7	...	...	...	...	...	
3 9	...	...	...	1	...	
3 14	...	...	...	1	...	
3 15	...	...	...	1	...	
3 20	...	...	...	1	...	
3 20	...	...	...	1	...	
Interval 3 20–3 42						
3 43	1	...	...	...	...	
3 45	1	...	...	...	...	
3 47	...	...	...	...	1	
3 50	...	...	...	1	...	

Time.	A 1	A 2	A 3	B	C	Notes.
3 50	...	1	...	...	...	
3 51	...	...	...	1	...	
3 55	1	...	...	...	...	
3 55½	1	...	...	...	...	
3 56	...	...	...	1	...	
3 58	1	...	...	...	...	
4 0	...	...	1	...	...	
4 0½	...	...	...	1	...	
4 2	1	...	...	...	...	
4 6	1	...	...	...	...	
4 10	...	...	...	1	...	
4 15	...	...	...	...	1	
4 15	...	...	...	1	...	Long course from west to east, nearly parallel to horizon, across Hydra. (?) Sluggish motion and faint train; manifestly not a Leonid.
4 22	1	...	...	...	...	
4 23	1	...	...	...	...	
4 25	...	...	...	2	...	Two faint meteors simultaneously giving downward near δ Leonis Maj.
4 30	...	...	1	...	...	
4 36	1	...	...	...	...	
4 48	1	...	...	...	...	
4 52	...	...	...	1	...	
4 59	...	...	1	...	...	
5 1	1	...	...	...	...	
5 4	...	1	...	...	...	
5 9	1	...	...	...	...	

Ceased observations at 5 20.

A large proportion of the meteors, including most of those that did not come from *Leo*, were faint. About ten seem to have been noted as having brilliant trains; all of these were from the direction of the Sickle. The flight of the meteors in all these cases was very swift. The trains appeared to G. C. T. generally to have a slight greenish tinge, and to give the impression of being broadest in the middle of the course, tapering towards each end.

Up to about 2 A.M. there was a strong moonlight with some amount of light cloud (not persistent), and perhaps some auroral light. After about 2 A.M. the sky was beautifully clear, without moonlight.

Real Paths of 107 *Meteors observed during the Ten Years ending* 1896 *November.* By W. F. Denning.

The following are the heights, radiants, &c. of such fireballs and shooting stars as have been observed at two or more stations in England during recent years, and submitted to comparison and calculation by the writer. Some of the results have been published before in a detached form in the Journals of the Liverpool Astronomical Society and British Astronomical Association, and in *The Observatory* or *Nature*. They are now brought together in a complete form, and arranged according to the day of the year. It is thought this plan will facilitate reference and prevent troublesome searches amongst various publications on the part of those making inquiries into the subject. Many of the observations were made in connection with the meteoric sections of the two societies above named. The principal observers were Mr. H. Corder, Bridgwater; Mr. D. Booth, Leeds; Professor A. S. Herschel, Slough; and the writer, Bristol. Several other observers contributed useful observations, and the abbreviations are :—

T. W. B.	T. W. Backhouse, Sunderland.
E. R. B.	E. R. Blakeley, Dewsbury.
D. B.	D. Booth, Leeds.
H. C.	H. Corder, Bridgwater.
G. T. D.	G. T. Davis, Reading.
W. F. D.	W. F. Denning, Bristol.
J. E.	J. Evershed, Kenley, Surrey.
A. S. H.	A. S. Herschel, Slough.
S. A. S.	S. A. Saunder, Wokingham, Berks.

In the column headed "Velocity" "v." means very, "sw." swift, "m." moderate or medium speed.

The greatest height of any well-observed meteor in the list was that of a small Perseid (No. 65) seen before sunrise on 1893 August 15, which at its first appearance was 126 miles above the Earth's surface.

The height of ordinary meteors is seldom above 100 miles. There are, however, some notable exceptions. A meteor of the first magnitude was seen at the same time on 1890 December 9 by Mr. Booth at Leeds and by myself at Bristol, and the observations, which appear quite consistent, indicate a height of 208 miles at first appearance and 165 at disappearance, but the figures are so exceptional when compared with others that I have, in order to be on the safe side, rejected the observation.

The average values derived from all the results summarised in the table are :—

Height at first appearance	73·6 miles	(106 meteors)
„ disappearance	45·3 „	(107 „)
Length of Path	62·1 „	(105 „)
Velocity	26·9 „	(58 „)

It must be remembered that these values are from a promiscuous collection of fireballs and shooting stars. Fireballs usually penetrate much lower into the atmosphere than small meteors, and they exhibit a longer flight and slower motion.

Catalogues of the real paths of meteors are not without their uses, as they supply data for after comparison. When one of these objects is seen, and its course determined from several independent observations, it is interesting to see if it accords with any meteor or meteor-shower noticed in preceding years. Thus on 1896 September 10, $9^h\ 3^m$, a brilliant meteor was recorded by many persons in England, and a comparison of the paths showed that it was directed from a radiant at 72°+42° in *Auriga* (No. 72). Now referring to Professor G. von Niessl's Catalogue we find two fireballs as follow :—

No. 78, 1866 Sept. 6	=☽.	Radiant 79°+44°.
No. 79, 1869 Sept. 8	4×☿.	Radiant 70+45.

The inference is therefore that the three objects belonged to one and the same stream. Extending the comparison to meteor showers we shall find the following radiants exhibit a good accordance in date and position :—

Tupman, No. 66, 1870, Aug. 29	75° + 45°
Denning, No. 520, 1885, Sept. 12–15	73 + 43
„ No. 549, 1879, Sept. 15–16	72 + 41

The mean positions of the radiants of the three fireballs and of the three meteor showers are as follow :—

Fireballs, Sept. 6–10	73°·7 + 43°·7
Meteor showers, Aug. 29–Sept. 16.	73·3 + 43·0

The agreement could scarcely be more satisfactory.

Such materials are also valuable for the purpose of investigating long-continued radiants, or radiants which, like that of the *Perseids*, shift their position amongst the stars. Thus Nos. 8 and 13 showed radiants nearly identical in *Cepheus*, while Nos.

17, 27, 35, &c., are near together in *Libra*. Among such resemblances some may be purely accidental, but the agreements in certain cases are so striking, and particularly those found in the radiants of meteor showers, that they deserve careful attention.

The apparent radiants of fireballs are often not so accurate as could be wished, owing to the nature of the observations which are more or less imperfect and incorrect. Happily, a decided improvement has been shown in recent years both in the number and quality of such records. Though a vast number of brilliant fireballs have been allowed to escape suitable investigation in the past, it is hoped the time will soon arrive when most of these bodies, which appear over Europe and America, will be sufficiently well observed to enable their real paths to be calculated with precision.

Bristol: 1896 *December* 6.

107 *Real paths of Fireballs and Shooting Stars observed in England from* 1886 *to* 1896. By W. F. Denning.

No.	Year.	Month and Day.	G.M.T.	Mag.	Height at Appearance.	Height at Disappearance.	Length of Obs. Path.	Velocity.	Radiant Point.	Observers.	Notes.
									α δ		
			h m		miles	miles	miles		° °		
1	1894	Jan. 1	10 9	2–1	50	34	20	v. sw.	107+25	H. C. and W. F. D.	Left a streak.
2		1	10 44	2	80	77	52	52	185+15	A. S. H. and W. F. D.	Radiant just risen.
3	1895	1	6 43	3–2½	58	47	30	13	233+56	H. C. and W. F. D.	Quadrantid.
4		1	7 23½	1½–1	65	50	41	10	62–12	,, ,,	Left a train of yellow sparks.
5	1894	2	9 40	3–2	65	56	53	21	240+48	A. S. H. and H. C.	Nucleus orange yellow.
6	1888	2	10 58	♃	98	60	109	44	250+57	D. B. and W. F. D.	Radiant $\eta-\theta$ Draconis.
7	1887	19	11 19	2–1	56	28	33	22	174+70	T. W. B. and W. H. S. Monck	Radiant at λ Draconis.
8	1894	25	10 1	☽	89	16	160	18	331+55	W. F. D. and others	Detonated, 45 observations.
9		Feb. 8	0 28	☽	80	20	...	sw.	Hercules	A. A. Rambaut and others.	In bright sunshine.
10		21	7 18	♃	...	30	...	sw.	141+52	W. F. D. and others	Not well observed.
11	1887	26	11 24	4	56	50	21	21	302+53	J. E. and W. F. D.	Radiant near ψ Cygni.
12	1896	Mar. 1	8 32	3–1½	55	55	405	20	18+5	T. W. B. and J. E. Clark	A very long flight. Radiant near horizon.
13	1895	10	14 27	♃–♀	50	23	58	19	330+59	H. C., A. Mee, and others	Seen during a total eclipse of the Moon, 6 obs. of No. 13.
14		10	15 24	♃	80	40	41	sw.	240+63	G. T. D. and others	Seen during a total eclipse of the Moon, 6 obs. of No. 13.
15	1896	16	10 13	♀	33	29	63	m.	205–18	R. B. Saul and others	Compare No. 17.
16	1887	27	8 49	1	59	55	23	19	11+35	T. W. B. and W. F. D.	Left a bright train.

No.	Year.	Month, and Day.	G.M.T.	Mag.	Height at Appearance.	Height at Disappearance.	Length of Obs. Path.	Velocity.	Radiant Point.	Observers.	Notes.
			h m		miles	miles	m l s		α δ		
17	1896	Apr. 8	8 20	1 – ♀	65	38	161	20	° ° 204 – 9	H. C., J. E., S. A. S., and others	Compare No. 15.
18		12	8 6	☽	118	34	177	19	50 + 42	F. W. Dyson and others	Radiant perhaps at 50 + 31.
19	1895	14	11 44	1	87	71	107	49	316 + 31	A. S. H. and W. F. D.	Compare No. 24.
20	1893	15	9 52	☽	99	40	158	32	15 + 59	Many Observers	Inexact.
21	1895	19	10 59	1 – ♀	91	43	97	33	269 + 30	H. C., E. R. B., W. F. D., and others	A fine Lyrid.
22		19	11 46	1	77	70	40	50	300 + 20	H. C. and W. F. D.	Left an orange streak.
23	1893	20	11 56	3 – 1½	67	48	28	24	198 + 5	A. S. H. and W. F. D.	Compare No. 33.
24		20	12 39½	2 – 1	75	68	35	49	314 + 27	,, ,,	Compare No. 19.
25		20	12 44	4 – 3	77	68	23	sw.	292 + 17	H. C. and W. F. D.	Left a streak.
26	1889	20	10 16	2 – 1	51	46	22	29	301 + 36	A. S. H. and T. W. B.	Radiant γ Cygni.
27		21	10 16	2 – 1	72	54	42	28	218 – 5	A. S. H. and W. F. D.	Radiant μ Libræ.
28	1893	21	13 38	5 – 3	62	45	37	slow	226 + 11	H. C. and W. F. D.	Radiant near β Libræ.
29		21	15 15½	3 – 2	80	65	33	v. sw.	299 ± 0	,, ,,	Left a streak.
30	1894	22	7 36	☽	80	17	120	30	α – β Persei	J. E. and others	Inexact. In twilight, 20 obs.
31	1887	May 8	8 22	☽	70	14	110	18	191 – 5	Many observers	Left a bright train.
32		9	10 27	3	73	50	37	slow	234 + 9	D. B. and W. F. D.	Radiant close to α Serpentis.
33	1893	9	11 21	♃	76	40	54	13	202 + 6	S. A. S. and W. F. D.	Compare No. 23.
34	1889	22	19 8	1 – ♃	50	58	292	14	63 + 35	G. T. D. and W. F. D.	Radiant on horizon.

No.	Year.	Month and Day.	G.M.T.	Mag.	Height at Appearance.	Height at Disappearance.	Length of Obs. Path.	Velocity.	Radiant Point.	Observers.	Notes.
									α δ		
			h m		miles	miles	miles		° °		
35	1889	May 29	10 44	☽	59	22	75	8	215−7	D. B. and I. W. Ward	Compare Nos. 27 and 37.
36	1895	June 10	9 59	☽	58	44	89	18	259−23	D. B. and others	Broke into three fragments.
37		July 7	10 49	☽	53	30	79	9	217−6	F. W. Dyson and others	Compare Nos. 27 and 35.
38	1891	13	12 54	>1	82	50	154	...	58+33	D. B. and J. Sherwen	Left a bright streak.
39	1887	19	10 48	3−2	59	55	18	slow	104+52	J. E. and W. F. D.	Left a streak.
40		21	11 24½	4	65	51	18	sw.	291+47	,, ,,	Radiant near δ Cygni.
41		22	13 12½	1−4	80	40	41	27	307+44	D. B. and W. F. D.	Radiant near α Cygni.
42	1886	Aug. 4	10 40	☽	90	11	168	21	161+59	W. F. D. and others	A brilliant Ursid fireball.
43	1888	5	10 18½	3−1	69	50	41	slow	50+55	G. T. D. and W. F. D.	Perseid. Left a streak.
44		5	10 29½	4−3	69	48	38	sw.	39+57	,, ,,	,, ,,
45		5	10 41½	4−3	68	48	41	sw.	43+51	,, ,,	,, ,,
46	1896	6	10 6	1	75	49	47	32	42+61	H. C. and W. F. D.	Perseid.
47	1895	7	10 12	3−2	43	28	42	sw.	45+47	,, ,,	Radiant near α Persei.
48		7	11 4	1−4	74	45	33	m.	333+36	A. S. H., H. C., and W. F. D.	A bright Lacertid.
49		7	11 28	2	105	63	64	sw.	38+57	H. C. and W. F. D.	Perseid. Left a streak.
50	1888	7	10 6	5−3	70	59	28	sw.	66+56	G. T. D. and W. F. D.	Camelid. ? Perseid.
51		8	10 10	4−3	65	52	38	v. sw.	318−13	,, ,,	A small Aquarid.
52		8	10 21	3	43	28	26	v. sw.	40+60	,, ,,	Perseid. Left a streak.

No.	Year.	Month and Day.	G.M.T.	Mag.	Height at Appearance.	Height at Disappearance.	Length of Obs. Path.	Velocity.	Radiant Point.	Observers.	Notes.
			h m		miles	miles	miles		α δ		
53	1888	Aug. 8	10 27½	4	68	48	36	sw.	° ° 42 + 57	G. T. D. and W. F. D.	Perseid. Left a streak.
54		8	11 16½	1	71	61	22	sw.	60 + 48	T. W. B. and D. B.	Radiant near μ Persei.
55	1887	10	10 47	2	111	43	120	sw.	44 + 54	T. W. B. and W. F. D.	Perseid. Left a streak.
56	1893	10	11 0	1	38	23	35	7	304 − 13	H. C. and W. F. D.	Motion very slow.
57		10	11 55½	1 − ♃	107	60	69	sw.	43 + 59	,, ,,	Perseid. Left a streak.
58	1894	10	11 35	1 − ♀	52	16	54	sw.	43 + 58	T. W. B. and E. R. B.	,, ,,
59	1896	10	10 6½	2	64	46	35	sw.	44 + 58	S. J. Johnson and W. F. D.	,, ,,
60		10	10 6¾	2	65	52	27	sw.	44 + 58	,, ,,	,, Inexact.
61	1895	11	10 7	4	78	62	30	43	48 + 60	A. S. H. and W. F. D.	,,
62		11	10 58½	1	95	61	53	53	36 + 57	A. S. H. and H. C.	,, Left a streak.
63		11	11 42½	♃	75	52	34	22	32 + 52	,, ,,	,, ,,
64	1888	13	11 33½	☽	79	47	48	sw.	43 + 56	D. B. and W. F. D.	A brilliant Perseid.
65	1887	14	13 46½	4 − 3	126	87	78	v. sw.	53 + 57	,, ,,	A Perseid at unusual height.
66	1893	14	11 0	1	60	43	15	slow	294 + 51	H. C. and W. F. D.	Cygnid. Very short path.
67		15	11 57½	♃	58	21	61	20	186 + 74	,, ,,	Radiant near χ Draconis.
68	1887	21	11 2	♀	80	45	40	13	264 + 61	D. B. and W. F. D.	,, ,, ξ ,,
69		21	11 25	4	71	51	26	34	35 + 63	,, ,,	,, ,, ϵ Cassiopeiæ.
70	1894	26	10 20	☽	90	30	66	sw.	305 + 79	H. C. and others	Streak lasted 30^m.

No.	Year.	Month and Day.	G.M.T. h m	Mag.	Height at Appearance. n.miles	Height at Disappearance. miles	Length of Obs. Path. miles	Velocity.	Radiant Point. α δ	Observers.	Notes.
71	1888	Aug. 26	8 18	☽	66	41	25	sw.	278°+52°	D. B. and W. F. D.	Left a train.
72	1896	Sept. 10	9 3	< ☽	82	65	78	31	72+42	T. E. R. Phillips and many others	Streak lasted $\frac{1}{2}^m$, 7 obs.
73		10	10 26	☽	71	30	44	15	330+71	H. C., W. F. D., and others	Radiant near β Cephei, 15 obs.
74	1888	12	10 $2\frac{1}{2}$	1	61	38	24	v. sw.	301+46	G. T. D. and J. E.	,, ,, δ Cygni.
75	1889	25	8 5	1	46	46	100	$8\frac{1}{2}$	244−22	W. H. Robinson and W. F. D.	,, on WSW. horizon.
76	1891	30	10 $2\frac{1}{2}$	♃ − ♀	54	24	56	13	14+7	D. B. and W. F. D.	,, at ϵ Piscium.
77	1893	Oct. 1	9 40	☽	105	28	93	v. slow	265+65	H. C. and others	Inexact.
78	1887	13	10 25	2−1	69	50	26	sw.	73+61	D. B. and W. F. D.	Radiant in Camelopardus.
79		13	11 25	4−2	70	42	37	sw.	127+83	,, ,,	,, ,,
80		14	12 $5\frac{1}{2}$	5−4	64	40	26	m.	355+36	,, ,,	,, Andromeda.
81		15	14 $48\frac{1}{2}$	$3-1\frac{1}{2}$	89	61	39	m.	87+15	,, ,,	Orionid. Left a streak.
82		20	11 45	4	106	90	34	sw.	87+15	,, ,,	,, at unusual height.
83		20	12 55	$1\frac{1}{2}-1$	92	53	70	sw.	$87+13\frac{1}{2}$	,, ,,	,, Left a streak.
84		20	14 15	2−1	74	41	43	slow	20+22	,, ,,	Radiant near ϕ Piscium.
85	1893	Nov. 12	6 53	1 − ♃	66	36	91	18	40+6	H. J. Townshend and W. F. D.	,, ,, α Ceti.
86	1896	12	11 8	< ♀	52	29	35	...	45+4	G. T. D. and others	Not very certain.
87		12	15 4	> 1	84	54	41	41	66+18	A. S. H. and H. C.	A bright Taurid.
88		12	15 44	1 − ♀	100	71	33	v.sw.	138+34	A. S. H. and H. C.	Same radiant as No. 97.
89	1888	13	17 19	> ♃	65	37	34	sw.	149+25	T. W. B. and W. F. D.	A fine Leonid. Streak for 9^m
90	1895	14	6 20	♂	49	29	40	slow	319−9	H. C., G. T. D., and S. A. S.	A very late Aquarid.

No.	Year.	Month and Day.	G.M.T. h	m	Mag.	Height at Appearance. miles	Height at Disappearance. miles	Length of Obs. Path. miles	Velocity.	Radiant Point. α	δ	Observers.	Notes.
91	1896	Nov. 14	13	54	>1	95	61	85	63	150°	+22°	A. S. H. and G. C. Thompson	A bright Leonid.
92		14	14	1½	♄	85	49	70	46	148	+20	A. S. H. and G. C. Thompson	A bright Leonid.
93		14	14	29	1 − ♂	92	55	60	v.sw.	150	+23	A. S. H. and H. C.	A bright Leonid.
94		14	17	27	1 - ♃	100	71	33	33	149	+24	H. C. and W. F. D.	Leonid.
95*		14	17	45	3	75	50	29	58	150	+24	A. S. H., H. C., and W. F. D.	Leonid.
96		14	17	45	3 − 2	127	89	45	45	150	+22	,, ,,	,, Doubtful.
97		14	18	5	3 − 2	70	55	16	32	133	+32	,, ,,	Very short path.
98	1886	17	7	18	♀	96	21	123	17½	34	+19	W. F. D. and others.	A bright Arietid.
99	1891	22	6	55	>1	60	42	47	9½	57	+15	D. B. and H. C.	Taurid. Compare next.
100	1895	22	6	50	☽	34	17	45	22	58	+22	A. S. Williams and others	A Taurid fireball.
101	1886	Dec. 4	9	7	♀	65	28	48	v. sw.	162	+58	W. F. D. and others	Left a streak 1½m. See No. 42.
102	1888	8	13	24	1	91	76	72	34	179	+10	T. W. B. and D. B.	Left a bright streak.
103	1892	9	7	39	1	91	57	75	30	103	+38	H. C. and W. F. D.	Geminid. Left a bright streak.
104	1895	11	14	59	1 − ♃	53	18	63	v. slow	50	+30	H. C. and T. H. Foulkes	Radiant W of ξ Persei.
105	1892	12	10	52	>1	60	36	31	20	110	+35	A. S. H., H. C., and W. F. D.	Geminid. Left a bright streak.
106		12	11	20	2	85	44	50	38	111	+34	H. C. and W. F. D.	Geminid.
107	1886	28	10	28	1	97	39	62	20	77	+30	W. F. D. and another	Left a bright train.

* There appear to have been several small Leonids at about $17^h\ 45^m$. No. 95 is, however, an excellent accordance depending upon three good observations. No. 96 is doubtful, and ought perhaps to be rejected.

Catalogue of Real Paths of Large Meteors.

By Professor Gustav von Niessl.

(*Communicated by W. F. Denning.*)

The following real paths of large meteors have been computed by me in recent years.

The descriptions of the objects were partly collected by myself, partly taken from various scientific papers and journals, viz. from Heis's *Wochenschrift für Astronomie*, *Reports of Vienna Academy*, the *Comptes Rendus* of the French Academy, the *American Journal of Science*, and others.

The succession of the several phenomena contained in this list is arranged according to the month and day (beginning with the commencement of the year), without regard to the particular years in which they were observed, as in my opinion this method of tabulation will allow a ready and convenient means of reference.

I have not availed myself of the very numerous and valuable notices and researches in the reports of the Luminous Meteor Committee of the British Association for the Advancement of Science (1848 to 1880), as one may hope that a summary of them will be prepared by Professor Herschel.

It is also to be hoped that a general catalogue of the real paths of fireballs will soon be compiled, and that this will include the results of other workers in this field; for example, Galle, Weiss, Newton, Koerber, &c.

Brünn, Austria:
1896 *November* 12.

List of Paths of Large Meteors &c., Computed by Professor G. von Niessl.

(*Communicated by W. F. Denning.*)

Ref. No.	Date of Appearance.	Time, G.M.T. and (Local Time).	Locality.	Apparent Size or Magnitude.	Height. When First Seen.	Height. When Last Seen.	Real Length of Observed Path.	Velocity to the Earth (Geocentric).	Radiant Point.		Number of Observations Used.	Remarks, References, &c.
		h m			English Miles.				α	δ		
1	Jan. 12 (1879)	6 26 (7 23)	Austria	= ☾ *	41	10	124	17	133°	+19°	6	*Thus: like Moon. Detonating.
2	12 (1879)	6 33 (7 30)	Austria	Large fireball	87	18	124	13	52	−10	6	Detonating.
3	16 (1895)	10 49 (11 49)	Austria	½ ☾	118	34	124	49	196	+56	5	Velocity not accurate.
4	16 (1895)	10 50 (11 50)	Austria	☾	35	31	166	47	172·5	−23·0	24	Detonating.
5	16 (1895)	10 52 (11 54)	Austria, Germany	☾	96	21	177	19	341·1	+56·4	26	Detonating.
6	17 (1890)	3 42 (4 40)	Austria	Brilliant	88	51	265	...	109	+23·5	2	
7	17 (1890)	4 6 (5 9)	Austria-Hungary	½ ☾ − ☾	97	25	353	34	113·6	+21·7	24	Detonating.
8	23 (1876)	4 44 (5 49)	Austria-Hungary	⅓ ☾	64	16	128	16	17	−20	5	
9	25 (1895)	6 51 (7 53)	Austria	About = ☾	97	31	83	23	104	+30	27	Detonating.

Ref. No.	Date of Appearance.	Time, G.M.T. and (Local Time).	Locality.	Apparent Size or Magnitude.	Height. When First Seen.	Height. When Last Seen.	Real Length of Observed Path. English Miles.	Velocity to the Earth (Geocentric).	Radiant Point. α	Radiant Point. δ	Number of Observations Used.	Remarks, References, &c.
		h m							°	°		
10	28 (1870)	5 31 (5 58)	Germany	3 × ♀	51	31	64	...	19·5	−20	2	See accounts in Heis's *Wochenschrift für Astronomie*, 1870.
11	Feb. 3 (1856)	7 55 (8 20)	Switzerland, France	Very large	138	48	248	62	120	− 7	3	See Heis's investigations in *Jahn, Unterhaltungen*, T. x.
12	3 (1882)	2 7 (3 45)	Hungary	Very large	115	5	297	...	264	+40	4	The Mocs Meteorites. Detonating.
13	10 (1875)	5 51 (6 0)	France	Of a singular magnitude	119	...	...	...	53	+50	4	Only radiant well determinable. *Comptes Rendus*, T. lxxx.
14	12 (1875)	15 45 (10 30)	United States	More than ☾	37	2·5	115	11	104	−30	9	The Marengo (Amana) Meteorites. See Leonard's account in *Americ. Journ. of Science and Arts*, Ser. 3, Vol. x. Detonating.
15	15 (1865)	5 29 (6 0)	Germany	☾	115	28	193	24	279	+60	4	See accounts in Heis's *Wochenschrift*, 1865.
16	20 (1870)	9 33 (10 5)	Italy	Bright meteor	46	23	69	23	273	+62	2	See accounts in Heis's *Wochenschrift*, 1870.
17	24 (1871)	9 9 (10 14)	Austria	♀	90	33	87	44	210	+55	2 (by Meteoroscopes)	
18	Mar. 4 (1872)	9 29 (10 1)	Italy	♃	62	36	37	37	115	+83	3	

Ref. No.	Date of Appearance.	Time, G.M.T. and (Local Time).	Locality.	Apparent Size or Magnitude.	Height. When First Seen.	Height. When Last Seen.	Real Length of Observed Path. English Miles.	Velocity to the Earth (Geocentric).	Radiant Point. α	Radiant Point. δ	Number of Observations Used.	Remarks, References, &c.
		h m							°	°		
19	9 (1822)	15 0 (10 0)	United States	⊙	41	29	76	40	257	+40	Many	See Dean's accounts in *Gilbert's Annalen*, T. lxxv., and *Poggendorf*, T. ii. Detonating.
20	13 (1883)	6 22 (7 21)	Austria	Large fireball	102	24	117	39	149	− 9	8	
21	17 (1871)	10 40 (10 49)	France	Large	115	...	more than 550	more than 18	345	+50	8	See accounts in *Comptes Rendus*, T. lxxii.
22	21 (1877)	5 59 (7 4)	Austria, Italy	> ♀	184	18	600	15	55	− 9	4	Scarcely accurate.
23	25 (1873)	7 30 (8 20)	Denmark	½ ☾	80	23	60	24	175	+55	4	See accounts in Heis's *Wochenschrift*, T. 1873.
24	Apr. 2 (1852)	6 25 (6 30)	France	Splendid meteor	10	9	83	12	215	+17	2	See Petit's investigations in *Comptes Rendus*, T. xxxv.
25	2 (1891)	7 50 (8 57)	Austria	½ ☾ − ☾	109	17	180	24	29·0	+55·2	28	Detonating.
26	9 (1876)	7 15 (8 38)	Austria-Hungary	☾	101	29	197	26	17	+57	7	Detonating.
27	10 (1874)	7 4 (8 4)	Austria	2 × ⊙	48	18	56	14	19	+57	6	Detonating.
28	11 (1869)	10 3 (10 35)	Italy	Bright meteor	96	57	40	...	207	+49	3	

Ref. No.	Date or Appearance.	Time, G.M.T. and (Local Time).	Locality.	Apparent Size or Magnitude.	Height. When First Seen. English	Height. When Last Seen.	Real Length of Observed Path. Miles.	Velocity to the Earth (Geocentric).	Radiant Point. α	Radiant Point. δ	Number of Observations Used.	Remarks, References, &c.
		h m							°	°		
29	11 (1871)	9 14 (9 46)	Italy	Bright meteor	...	25	...	...	216	−10	3	
30	21 (1887)	8 0 (8 58)	Austria	☾	82	23	223	25	214	−13	15	Detonating.
31	22 (1871)	10 5) (10 37)	Italy	Bright meteor	...	43	...	...	231	− 7	2	Radiant, not accurate, but probably not identical with the former of Apr. 21.
32	22 (1888)	6 52) (7 55)	Austria	¾ ☾	116	23	186	20	100·7	+ 10·1	44	
33	29 (1877)	8 37 (10 17)	Sweden	☾	42	19	73	21	146·5	± 0	4	See Sundell's account in *Finska Vetenskaps Societ. Förhandl.* T. xxvii.
34	May 5 (1869)	6 2 (6 32)	Germany	Large	30	5	48	24	190	+ 8	3	The meteorite of Krähenberg. See *Neumayer in Vienna Acad.* T. lx. Detonating.
35	14 (1864)	8 0 (8 0)	France	☾	174	14	448	19	86·5	+ 24·0	11	The meteorites of Orgueil.
36	20 (1869)	16 17 (11 25)	United States	Very large	53	28	46	...	240	−18	3	See Loomis's accounts in *Amer. Jour.* Ser. II. Vol. xlviii. Detonating.
37	26 (1751)	4 56 (6 0)	Austria-Hungary	☾	118	28	280	...	60	+ 20·5	2	Ironfall at Hradschin. Detonating.
38	28 (1880)	7 26 (8 39)	Austria-Hungary	¼ ☾	51	27	74	12	100	+ 35	2	

Ref. No.	Date of Appearance.	Time, G.M.T. and (Local Time).	Locality.	Apparent Size or Magnitude.	Height. When First Seen.	Height. When Last Seen.	Real Length of Observed Path. English Miles.	Velocity to the Earth (Geocentric).	Radiant Point. α	Radiant Point. δ	Number of Observations Used.	Remarks, References, &c.
		h m							°	°		
39	June 2 (1886)	9 59 (11 0)	Austria	> ♃	...	25	...	...	250	−20	4	Not accurate.
40	3 (1842)	8 51 (9 0)	France	☉	130	11	165	30	272	+46·5	5	See Petit's accounts in *Comptes Rendus*, T. xiv. and xvi. Detonating.
41	3 (1883)	8 44 (9 50)	Austria	Splendid meteor	124	21	332	22	249·9	−20·2	11	
42	3 (1883)	10 39 (10 57)	North Sea	☾	188	23	454	23	248	−20	6	
43	7 (1858)	9 29 (10 0)	Germany	Brilliant meteor	85	18	115	53	256	+ 2	5	See Heis's accounts in *Wochenschrift*, 1858.
44	8 (1888)	17 24 (18 30)	Germany	Large	28	4	86	...	90	+28	3	= June 9, 6^h 30^m A.M. in full sunshine. Detonating.
	(1841)	8 13 (8 22)	France	> 1	115	87	468	43	266	−16	3	See Petit's account in *Comptes Rendus*, T. xiii.
46	9 (1866)	3 32 (4 56)	Austria-Hungary	☉	36	7	29	...	170	+55	4	The *Knyahinya Meteorite*. Detonating.
47	9 (1869)	8 26 (9 7)	Germany	Large meteor	128	28	119	27	250	+35	7	See Heis's account in *Wochenschrift*, 1869.
48	11 (1867)	8 8 (8 11)	France Germany	Almost ☾	71	41	93	...	86	+44	6	
49	14 (1877)	8 43 (8 52)	France	Splendid	140	27	134	34	215	+14	3	See Grueys' investigations in *Comptes Rendus*, T. lxxxv. Detonating.

Ref. No.	Date of Appearance.	Time, G.M.T. and (Local Time)	Locality.	Apparent Size or Magnitude.	Height. When First Seen.	Height. When Last Seen	Real Length of Observed Path. English Miles.	Velocity to the Earth (Geocentric).	Radiant Point α	Radiant Point δ	Number of Observations Used.	Remarks, References, &c.
		h m							°	°		
50	17 (1868)	8 18 (9 23)	Austria	¼ ☾	...	32	...	...	109	+37	3	A rough determination.
51	17 (1873)	7 38 (8 45)	Austria-Hungary	½ ☉	92	20	281	28	248·6	−20·2	25	Detonating.
52	17 (1885)	8 47 (10 0)	Bosnia	☾	57	30	130	30	112	+42	6	
53	19 (1887)	9 23 (10 25)	Austria	Bright meteor	122	21	360	...	282	−19	4	
54	July 7 (1892)	8 2 (9 0)	South Europe	⅙ ☾	46	98	836	54	349	+ 8	21	True course upwards.
55	8 (1856)	11 38 (6 0)	United States	Large meteor	34	7	37	...	157	+15	4	See Spillman's accounts in *Amer. Journ.* Vol. xxii. and xxiii. Detonating.
56	8 (1876)	14 31 (8 45)	United States	Large	88	34	150	...	305	+ 6	...	See Kirkwood's investigations in *Proceed. of the Amer. Philosph. Soc.* 1877.
57	13 (1879)	7 30 (8 27)	Austria	⅓ ☾	60	19	119	25	246	−19	10	Detonating.
58	16 (1871)	6 45 (7 34)	Germany	10 × ♀	23	22	48	...	294	−15	2	See Heis's accounts in *Wochenschr*, 1871. Detonating.
59	17 (1876)	7 28 (8 33)	Austria	♀	102	49	55	18	260	+45	3	

Ref. No.	Date of Appearance.	Time, G.M.T. and (Local Time).	Locality.	Apparent Size or Magnitude.	Height. When First Seen.	Height. When Last Seen.	Real Length of Observed Path. English Miles.	Velocity to the Earth (Geocentric).	Radiant Point. δ	Radiant Point. α	Number of Observations Used.	Remarks, References, &c.
		h m							°	°		
60	20 (1860)	14 43 (9 35)	United States	Large meteor	126	42	1000 at least	12·5	159	+19	6	See Lyman's account in *Americ. Journ.* Vol. xxx.
61	22 (1888)	7 3 (7 55)	Adria	Bright	...	21	...	...	227	+21	4	Observations incomplete, determinations only rough.
62	26 (1873)	9 18 (10 23)	Austria-Hungary	♀	50	46	23	...	290	−29	2 (by Meteoroscopes)	
63	30 (1873)	8 31 (9 12)	Germany	> ♀	82	35	184	41	317	−11	2	See Heis's accounts in *Wochenschr.*, 1873.
64	30 (1879)	9 51 (10 42)	Denmark	♀	107	46	83	...	330	+22·5	2	See Kohl's accounts in Klein's *Wochenschrift*, 1879.
65	Aug. 2 (1860)	15 41 (10 5)	United States	Brilliant meteor	109	28	370	32·5	317	−23	8	See Prof. Newton's accounts in *Amer. Journ.* Ser. II. Vol. xxxiii. Detonating.
66	4 (1858)	9 9 (9 40)	Germany	½ ☾	78	28	70	14	208	+51	4	See accounts in Heis's *Wochenschr.*, 1858 and 1863. Detonating.
67	6 (1864)	10 20 (10 51)	Germany	> ♀	106	36	142	38	305	− 8	5	See Heis's accounts in *Wochenschr.*, 1867.
68	7 (1859)	7 59 (8 30)	Germany	Large meteor	138	27	110	24	264	+53	2	See Heis's accounts in *Wochenschr.*, T. ii. Detonating.
69	10 (1858)	8 32 (9 3)	Germany	⅔ ☾	128	50	92	23	215	+55	3	See Heis's accounts in *Astr. Nachr.* 1186.

Ref. No.	Date of Appearance.	Time, G.M.T. and Local Time).	Locality.	Apparent Size or Magnitude.	Height. When First Seen.	Height. When Last Seen	Real Length of Observed Path.	Velocity to the Earth. (Geocentric).	Radiant Point. α	Radiant Point. δ	Number of Observations Used.	Remarks, References, &c.
					English Miles.							
70	10 (1863)	h m 8 55 (9 45)	Italy	♀	73	18	...	...	30	+70	2	Not a Perseid.
71	12 (1880)	10 44 (11 34)	Norway and Denmark	♀	79	37	166	17	257·5	− 3	3	See Kohl's investigations in Klein's *Wochenschr.*, 1880.
72	17 (1859)	8 17 (8 53)	Germany	½ ☾	83	21	119	43	335	+17·0	5	See Heis's accounts in *Wochenschr.* T. iii. Detonating.
73	19 (1847)	9 14 (9 23)	France	2 × ♃	108	22	266	23	253	−16·5	2	See Petit's accounts in *Comptes Rendus.* T. xxv. and T. xxix.
74	19 (1867)	8 37 (9 18)	Germany	3 × ♃	158	27	133	30	279	+43	3	See Heis' accounts in *Wochenschr.*, 1867.
75	25 (1884)	8 23 (9 25)	Austria	Very splendid	45	32	69	...	10	+ 5·5	2	
76	26 (1858)	18 45)	Germany	☾	44	41	64	41	11	± 0	2	See Heis's accounts in *Wochenschr.* T. ii. Rad. not accurate.
77	Sept. 5 (1868)	8 0 (8 0)	Servia to France	> ♃	483	115	1770	41	13·9	− 2·0	12	One of the longest paths.
78	6 (1866)	6 50 (7 55)	Austria	☾	37	32	73	...	79	+44	2	
79	8 (1869)	6 17 (7 22)	Austria, Italy	4 × ♃	115	46	734	73	70	+45	Many, but rough	

Ref. No.	Date of Appearance.	Time, G.M.T. and (Local Time).	Locality.	Apparent. Size or Magnitude.	Height. When First Seen.	Height. When Last Seen.	Real Length of Observed Path.	Velocity to the Earth (Geocentric).	Radiant Point α	Radiant Point δ	Number of Observations Used.	Remarks, References, &c.
		h m			English Miles.				°	°		
80	19 (1873)	10 38 (9 10)	Germany	☾	69	23	106	21	208	+50	2	See accounts in Heis's *Wochenschr.*, 1873.
81	27 (1870)	5 27 (6 8)	Germany, Denmark	Large meteor	184	10	450	44	150	+30	18	See a discussion of Dr. Matthiesen's account in *A.N.*, 1845 and 1846.
82	Oct. 12 (1856)	5 5 (6 0)	Austria	Splendid meteor	73	18	110	...	224	+25	3	
83	13 (1879)	5 0 (5 51)	Italy	2 × ♃	138	20	220	13	304·5	−11	5	
84	23 (1805)	6 22 (6 58)	Germany	2 × ♀	55	37	101	29	52	+16·5	3	See accounts in *Gilbert's Annalen*. T. xxiii.
85	23 (1887)	3 19 (4 20)	Austria	☾	136	20	322	30	224	− 8	10	Detonating.
86	23 (1889)	4 17 (5 29)	Austria-Hungary	¼ ☾	105	23	168	14	311·4	−11·3	9	
87	29 (1857)	5 57 (6 6)	France	Large meteor	83	37	179	23	231	+ 6	2	See Petit's accounts in *Astr. Nach.*, 1180.
88	Nov. 1 (1857)	5 55 (6 26)	Germany	⅕ ☾	157	13	161	...	294	+42	2	
89	11 (1864)	5 35 (6 8)	France, Germany	½ ☾	106	30	446	68	55	+21	6	See accounts in Heis's *Wochenschr.*, 1864.
90	11 (1879)	4 44 (5 34)	Austria	Splendid meteor	50	46	11	...	271	−11	3	

Ref. No.	Date of Appearance.	Time, G.M.T. and (Local Time).	Locality.	Apparent Size or Magnitude.	Height. When First Seen.	Height. When Last Seen.	Real Length of Observed Path. English Miles.	Velocity to the Earth (Geocentric).	Radiant Point. α	Radiant Point. δ	Number of Observations Used.	Remarks, References, &c.
		h m							d	d		
91	13 (1873)	7 48 (8 30)	Germany	⅓ ☾	57	18	64	22	84	+51	2	See Heis's accounts in *Wochenschr.*, 1873.
92	Dec. 4 (1885)	6 32 (7 34)	Austria	Brilliant meteor	48	34	110	18	301·4	−12·8	2	
93	7 (1865)	7 40 (7 30)	France	< ☉	64	45	92	9	296	+11	4	See Gruey's accounts in *Comptes Rendus.* T. lxii. Detonating.
94	7 (1868)	3 30 (4 8)	Germany	♃	56	41	165	14	43	+ 4	3	See Heis's account in *Wochenschr.*, 1868.
95	13 (1888)	5 54 (6 52)	Austria, Germany	♀	85	19	136	27	78	+22	6	
96	17 (1857)	4 18 (4 54)	Germany	Large meteor	48	18	207	34	293	−20	4	See accounts in Heis's *Wochenschr.*, 1858.
97	21 (1887)	3 51 (5 0)	Austro-Hungary	☾	55	29	68	19	273	+19·5	5	
98	24 (1850)	6 18 (6 27)	France	Large meteor	50	24	27	5	0	+50·7	2	See Petit's account in *Comptes Rendus.* T. xlii. Detonating.
99	24 (1873)	12 47 (7 39)	United States	☾	87	9	230	34	109	+26	20	See account by Cleveland Abbe, Washington *Phil. Society's Transact.*, 1874. Detonating.
100	31 (1888)	7 40 (8 8)	France, Germany	♀	74	46	336	...	235	+42	3	See account in Klein's *Wochenschr.*, 1889.

Observations of Comet, 1896, *1. (Perrine-Lamp), at the Radcliffe Observatory, Oxford.*

(*Communicated by E. J. Stone, Esq., M.A., F.R.S., Radcliffe Observer.*)

The following comet observations (with the exception of March 4) were made with the 10-inch Barclay Equatorial, using the ring micrometer, with power 100. The observation on March 4 was made with the Heliometer, using the ring micrometer, with power 100.

Date. 1896.	G.M.T. h m s	Local Sidereal Time. h m s	Observer.	Comet *minus* Star (corrected for Refraction only). R.A. m s	N.P.D. ′ ″	No. of Comps.	Apparent R.A. of Comet. h m s	Parallax in R.A. *v*. s	Log. $(p \times \Delta)$.	Apparent N.P.D. of Comet. ° ′ ″	Parallax in N.P.D. *q*. ″	Log. $(q \times \Delta)$.	Ref.
Feb. 23	16 46 12	14 55 28	R.	−1 13·41	−8 33·5	5	21 22 25·31	−1·11	9·6340	57 22 23·3	14·1	0·7377	(*a*)
23	16 49 58	14 59 14	R.	−1 49·20	−8 55·6	3	21 22 29·39	−1·11	9·6348	57 21 39·4	14·2	0·7416	(*b*)
23	17 53 31	16 2 57	R.	−2 18·60	−6 9·0	2	21 23 30·33	−1·11	9·6321	57 9 36·4	13·7	0·7259	(*c*)
Mar. 3	9 19 19	8 2 50	R.	+1 11·40	+1 21·3	8	1 6 0·30	+1·07	9·7572	38 14 58·2	6·0	0·5077	(*d*)
4	8 26 21	7 13 40	R.	+1 23·49	−0 47·2	8	1 24 57 31	+1·05	9·7702	38 14 34·4	7·2	0·6089	(*e*)
14	9 31 33	8 58 29	R.	−1 49·11	+6 8·1	9	3 14 33·50	+0·59	9·7334	42 16 48·2	4·8	0·6396	(*f*)
16	10 4 39	9 39 33	R.	−1 15·09	+2 25·5	8	3 25 42·52	+0·54	9·7270	43 6 39·0	4·6	0·6509	(*g*)
19	8 5 44	7 57 11	W.	+1 25·57	...	10	3 38 49·13	+0·43	9·6756	...	...	...	(*h*)
19	8 5 44	7 57 11	W.	+1 20·70	−5 58·8	10	3 38 48·71	+0·43	9·6756	44 11 9	2·9	0·4942	(*i*)
Apr. 9	9 58 36	11 8 6	R.	−3 34·04	−0 54·1	8	4 29 30·09	+0·26	9·6798	48 48 10·3	2·5	0·6617	(*j*)
10	9 19 31	10 32 51	R.	−0 51·33	−1 26·9	6	4 31 1·07	+0·26	9·6851	48 55 49·8	2·8	0·7140	(*k*)

Observers' Remarks.

(*a*), (*b*), (*c*) Coma is 2′ in diameter. Nucleus diffused, mag. 9 or 10. The comet is visible in the 3-inch finder.

(*c*) Very faint, strong twilight.

(*d*) The comet's coma is, approximately, 3′ in diameter. The nucleus (a large nebulous image, say 15″) is brighter than on February 23.

(*e*) Condensation bright, but more nebulous than with Barclay last night.

(*f*) Coma of comet 2′ diameter; condensation seemed stellar at times, mag. 10. The comparison star is reddish. Sky hazy.

(*g*) Coma 2′ in diameter, with a nebulous condensation. Nucleus visible at times, mag. 11. Sky hazy at intervals.

(*h*), (*i*) The comet is a round diffused mass, with a stellar brightening showing occasionally as a nucleus of about the 13th magnitude. A very small star was close to the comet's nucleus at about $8^h\ 10^m$ G.M.T. Moon near, but sky clear. Observing almost impracticable at times, noises.

(*j*) There is no nucleus, but a slight condensation shows up at times. Diameter of coma 1′. A bright star, *e Persei*, in the field.

(*k*) Observations rather difficult. Coma large, but feeble. A star, magnitude 12 or 13, follows within the condensation, which is only faintly visible. Wind slightly gusty, causing occasional small vibrations of the telescope.

Observers: W., Mr. W. Wickham; R., Mr. W. H. Robinson.

Assumed Places of Comparison Stars.

Ref.	Mean R.A.	Reduction to Apparent R.A.	Mean N.P.D.	Reduction to Apparent N.P.D.	Authority.
	h m s	s	° ′ ″	″	
(*a*)	21 23 39·67	−0·95	57 30 48·0	+ 8·8	Leiden A. G. Zone 90, No. 48, and Zone 100, No. 116.
(*b*)	21 24 19·54	−0·95	57 30 26·2	+ 8·8	Leiden A. G. Zone 90, No. 49, and Zone 100, No. 117.
(*c*)	21 25 49·89	−0·96	57 15 36·8	+ 8·6	Leiden A. G. Zone 90, No. 51, and Zone 100, No. 119.
(*d*)	1 4 50 09	−1·19	38 13 49·3	−12·4	Radcliffe Transit-Circle observations, 1896, September 23, November 18.
	1 23 34·86	−1·04	38 15 35·2	−13·6	Radcliffe Transit-Circle observations, 1896, October 21, November 6.
(*f*)	3 16 22·58	+0·03	42 10 57·4	−17·2	Bonn A. G. 2817.
(*g*)	3 26 57·49	+0·12	43 4 30·7	−17·1	Bonn A. G. 2999.
(*h*)	3 37 23·37	+0·19	...	...	Bonn A. G. 3125.
(*i*)	3 37 27·82	+0·19	44 17 24·6	−16·7	Bonn A. G. 3126. This star has proper motions; not applied.
(*j*)	4 33 3·73	+0·40	48 49 18·6	−14·3	Barclay Equatorial comparisons with Bonn A. G. 3748 and 3776.
(*k*)	4 31 52·02	+0·38	48 57 30·8	−14·1	Bonn A. G. 3748.

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·85; and the geocentric distances, Δ, are taken from the *Astronomische Nachrichten*, as follows: (*a*), (*b*), (*c*) No. 3335; (*d*), (*e*), (*f*), (*g*), (*h*), (*i*) No. 3338; (*j*), (*k*) No. 3341.

Radcliffe Observatory, Oxford: 1897 *January* 8.

Observations of Occultations of Stars and of Jupiter and his Satellites by the Moon and of Phenomena of Jupiter's Satellites made at the Royal Observatory, Greenwich, in the year 1896.

(Communicated by the Astronomer Royal.)

Day	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Feb. 25	Disapp. W.B. (2) VII. 1457	Sheepshanks Equat.	100	Dark	7 57 29·69	D.
25	,, ,,	Astrographic Equat.	120	,,	7 57 29·48	R. C.
25	,, ,,	Altazimuth	100	,,	7 57 29·72	S.
Mar. 1	Reapp. Piazzi XII. 196	Sheepshanks Equat.	100	,,	14 44 32·47	H.
19	Disapp. 18 Tauri	Altazimuth	100	,,	7 41 35·19	D.
	,, ,,	Sheepshanks Equat.	120	,,	7 41 33 26	H.
19	,, ,,	Finder of Astrographic Equat.	...	,,	7 41 34·76	J.
27	,, 79 Leonis	28-inch Equat.	670	,,	9 57 47·50	L.
27	,, ,,	Sheepshanks Equat.	100	,,	9 57 47·05	B.
27	,,	Astrographic Equat.	225	,,	9 57 47·45	D. E.
27	,, ,,	Altazimuth	100	,,	9 57 47·55	S.
27 (*a*)	Reapp. ,,	28-inch Equat.	670	Bright	10 55 0·45	L.
27 (*b*)	,, ,,	Transit Circle	195	,,	10 54 59·23	B.
27	,, ,,	Astrographic Equat.	225	,,	10 54 59·72	D. E.

Day.	Phenomenon.		Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1896.							
Mar. 27	Reapp.	79 Leonis	Sheepshanks Equat.	100	Bright	10 54 59·49	S.
Apr. 26	Disapp.	83 Virginis	Astrographic Equat.	225	Dark	9 53 48·45	J.
26 (*c*)	Reapp.	,,	,,	225	Bright	11 1 25·34	,,
May 18	Disapp.	W.B. (2) VIII. 1240	Sheepshanks Equat.	100	Dark	8 44 39·55	W. S.
21	,,	*v* Leonis	,,	100	,,	10 46 9·92	A. C.
21	,,	,,	Altazimuth	100	,,	10 46 10·31	S.
June 14 (*d*)	,,	Jupiter's Satellite IV.	28-inch Equat.	670	,,	9 40 47·19	D.
14	,,	,,	Sheepshanks Equat.	100	,,	9 40 46·80	A. C.
14	,,	,,	Detached Tel., No. 2	75	,,	9 40 47·86	B.
14	,,	,,	Altazimuth	60	,,	9 40 (43·15)	C. D.
14	,,	,,	Astrographic Equat.	120	,,	9 40 46·22	H. F.
14	,,	Jupiter's 1st Limb	28-inch Equat.	670	,,	9 51 54·85	D.
14	,,	,,	Corbett Tel.	100	,,	9 51 57·35	L.
14	,,	,,	Sheepshanks Equat.	100	,,	9 51 57·26	A. C.
14	,,	,,	Detached Tel., No. 2	75	,,	9 52 (1·05)	B.
14	,,	,,	Altazimuth	100	,,	9 51 59·30	C. D.
14 (*e*)	,,	,,	Astrographic Equat.	225	,,	9 51 57·49	H. F.
14	,,	,,	28-inch Equat.	670	,,	9 52 47·71	D.
14	,,	Jupiter's 2nd Limb	Corbett Tel.	100	,,	9 52 44·82	L.
14	,,	,,					

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1896. June 14	Disapp. Jupiter's 2nd Limb	Sheepshanks Equat.	100	Dark	9 52 47·12	A. C.
14	,, ,,	Detached Tel., No. 2	75	,,	9 52 47·93	B.
14	,, ,,	Altazimuth	100	,,	9 52 45·96	C. D.
14	,, ,,	Astrographic Equat.	225	,,	9 52 46·45	H. F.
14	,, Jupiter's Satellite I.	28-inch Equat.	670	,,	9 54 57·36	D.
14	,, ,,	Corbett Tel.	100	,,	9 54 56·36	L.
14	,, ,,	Sheepshanks Equat.	100	,,	9 54 57·56	A. C.
14	,, ,,	Detached Tel., No. 2	75	,,	9 55 (0·57)	B.
14	,, ,,	Astrographic Equat.	225	,,	9 54 57·00	H. F.
14	,, Jupiter's Satellite II.	28-inch Equat.	670	,,	9 55 32·96	D.
14	,, ,,	Corbett Tel.	100	,,	9 55 34·26	L.
14	,, ,,	Sheepshanks Equat.	100	,,	9 55 33·57	A. C.
14	,, ,,	Detached Tel., No. 2	75	,,	9 55 34·48	B.
14	,, ,,	Altazimuth	60	,,	9 55 32·71	C. D.
14	,, ,,	Astrographic Equat.	225	,,	9 55 32·30	H. F.
14	,, Jupiter's Satellite III.	28-inch Equat.	670	,,	9 57 56·56	D.
14	,, ,,	Corbett Tel.	100	,,	9 57 57·10	L.
14	,, ,,	Sheepshanks Equat.	100	,,	9 57 57·16	A. C.

Day. 1896.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
June 14	Disapp. Jupiter's Satellite III.	Altazimuth	60	Dark	9 57 56·81	C. D.
14	,, ,,	Astrographic Equat.	225	,,	9 57 55·30	H. F.
22	,, 4 Scorpii	Astrographic Equat.	120	,,	8 57 3·46	C. D.
24	,, Piazzi XVII. 359	,,	120	,,	11 56 8·51	,,
July 20	Reapp. τ Scorpii	Sheepshanks Equat.	55	Bright	8 16 31·17	A. C.
28 (*c*)	Disapp. 21 Piscium	Astrographic Equat.	225	,,	15 9 51·12	C. D.
28	Reapp. ,,	,,	120	Dark	16 8 54·95	,,
Oct. 1 (*c*)	Disapp. ϵ Cancri	Sheepshanks Equat.	100	Bright	16 40 0·47	W.
1 (*c*)	,, 42 Cancri	,,	100	,,	16 47 7·31	,,
22 (*c*)	,, μ Arietis	Astrographic Equat.	225	,,	7 45 29·15	H.
27	,, 52 Geminorum	,,	225	,,	10 9 58·15	W. B.
Dec. 17	,, η Tauri	Sheepshanks Equat.	100	Dark	4 13 41·59	H.
17 (*e*)	Reapp. 20 Tauri	,,	100	Bright	4 17 54·80	,,
17 (*e*)	,, η Tauri	,,	100	,,	4 38 17·40	,,

Notes.

(*a*) Observer noted that the observed time was probably late.

(*b*) The observed time has been increased by 5 seconds after consultation with the Observer, who was able to verify the alteration by remembering the position of the star at reappearance in the field at the Transit Circle, and to compare with the observed time of transit.

(*c*) Not considered a good observation. (*d*) The disappearance of the satellite seemed to last about a second.

(*e*) Seemed to be at a slight distance from limb when first seen.

Phenomena of Jupiter's Satellites.

Day.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation.	Mean Solar Time of *N.A.*	Observer.
1896.					h m s	h m s	
Jan. 17	I. (*a*)	Ecl. D. Began to fade	Altaz.	100	6 20 48		H. F.
17	I.	Bisection	,,	,,	6 22 43		,,
17	I.	Last seen	,,	,,	6 24 18	6 22 51	,,
22	I. (*b*)	Ecl. D. Began to fade	Sheepsh. Equat.	120	13 46 0		B.
22	I.	Bisection	,,	,,	13 47 24		,,
22	I.	Last seen	,,	,,	13 48 34	13 48 28	,,
29	III.	Ecl. R. First seen	,,	,,	7 45 36	7 43 25	H.
29	III.	Full brightness	,,	,,	7 48 36		,,
Feb. 6	II.	Ecl. R. First seen	Astrographic Equat.	225	7 47 37	7 48 3	D. E.
11	II. (*c*)	Tr. Ing. First contact	Sheepsh. Equat.	120	11 25 5		B.
11	II.	Bisection	,,	,,	11 29 14	11 28 0	,,
11	II.	Last contact	,,	,,	11 32 24		,,
22	I.	Tr. Ing. First contact	,,	200	12 30 2		H.
22	I.	Bisection	,,	,,	12 31 31	12 32 0	,,
22	I.	Last contact	,,	,,	12 33 56		,,
23	IV. (*d*)	Ecl. R. First seen	,,	120	6 19 52	6 22 27	A. C.
23	IV.	Full brightness	,,	,,	6 23 17		,,
25	I.	Ecl. R. First seen	Altaz.	100	7 9 18	7 8 17	D.

Day.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation.	Mean Solar Time of *N.A.*	Observer.
1896.					h m s	h m s	
Mar. 1	I.	Ecl. R. First seen	Sheepsh. Equat.	120	14 34 34	14 34 34	H.
1	I.	Full brightness	,,	,,	14 36 30		,,
10	I. (*e*)	Occ. D. First contact	,,	200	7 36 5		,,
10	I.	Bisection	,,	,,	7 39 0	7 41 0	,,
10	I.	Last seen	,,	,,	7 40 54		,,
16	II.	Ecl. R. First seen	,,	120	10 2 47	10 2 46	B.
16	II.	Full brightness	,,	,,	10 4 49		,,
19	III.	Occ. R. First seen	,,	100	7 10 16		H.
19	III.	Bisection	,,	,,	7 11 56	7 15 0	,,
19	III.	Last contact	,,	,,	7 13 16		,,
19	I.	Ecl. R. First seen	,,	,,	7 22 32	7 22 31	,,
19	I.	Full brightness	,,	,,	7 24 14		,,
19	III.	Ecl. D. Last seen	,,	,,	8 10 51	8 9 38	,,
19	III.	Ecl. R. First seen	,,	,,	11 42 3	11 41 42	,,
19	III.	Full brightness	,,	,,	11 45 51		,,
23	II.	Occ. D. First contact	,,	,,	7 25 48		,,
23	II.	Bisection	,,	,,	7 28 3	7 28 0	,,
23	II.	Last seen	,,	,,	7 30 47		,,

Day.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time of *N.A.* h m s	Observer.
1896.							
Mar. 25	I.	Tr. Ing. First contact	Sheepsh. Equat.	100	8 37 55		B.
25	I.	Bisection	,,	,,	8 41 40	8 40 0	,,
25	I. (*f*)	Last contact	,,	,,	8 45 14		,,
25	I. (*g*)	Tr. Egr. First contact	,,	,,	10 57 17		,,
25	I.	Bisection	,,	,,	11 0 32	11 0 0	,,
25	I.	Last contact	,,	,,	11 3 21		,,
27	IV.	Occ. R. Last contact	28-inch Equat.	670	7 11 48	7 9 0	L.
27	IV. (*h*)	Ecl. D. Last seen	Sheepsh. Equat.	120	13 55 36	13 53 42	B.
30	II. (*i*)	Occ. D. First contact	,,	,,	9 53 50		A. C.
30	II.	Bisection	,,	,,	9 55 31	9 57 0	,,
30	II.	Last seen	,,	,,	9 59 6		,,
Apr. 1	I.	Tr. Ing. First contact	,,	55	10 32 36		H. A.
1	I.	Bisection	,,	,,	10 34 45	10 31 0	,,
1	I.	Last contact	,,	,,	10 37 20		,,
2	I. (*j*)	Occ. D. Last seen	Sheepsh. Equat.	120	7 41 13	7 41 0	W.
2	III.	Occ. D. Bisection	,,	,,	11 7 55	11 6 0	,,
2	III.	Last seen	,,	,,	11 9 9		,,
2	I.	Ecl. R. First seen	,,	,,	11 13 24	11 13 14	,,
2	I.	Full brightness	,,	,,	11 14 39		,,

Day.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time of *N.A.* h m s	Observer.
1896.							
Apr. 3	I.	Tr. Egr. Last contact	28-inch Equat.	670	7 21 46	7 19 0	L.
10	I.	Tr. Egr. First contact	Sheepsh. Equat.	55	9 10 10		H. F.
10	I.	Bisection	,,	,,	9 12 19	9 12 0	,,
10	I.	Last contact	,,	,,	9 14 59		,,
13	IV.	Ecl. D. Began to fade	,,	120	7 47 3		H.
13	IV.	Last seen	,,	,,	7 59 56	7 56 58	,,
13	III.	Tr. Egr. First contact	,,	,,	8 34 46		H.
13	III.	Bisection	,,	,,	8 37 40	8 39 0	,,
13	III.	Last contact	,,	,,	8 42 29		,,
15	II.	Tr. Ing. First contact	28-inch Equat.	670	9 18 24		D.
15	II.	Bisection	,,	,,	9 21 4	9 23 0	,,
15	II.	Last contact	,,	,,	9 23 20		,,
15	II.	Tr. Ing. First contact	Corbett Telescope	100	9 15 49		C.
15	II.	Bisection	,,	,,	9 20 50	9 23 0	,,
15	II.	Last contact	,,	,,	9 23 20		,,
17	I. (*k*)	Tr. Ing. First contact	Sheepsh. Equat.	120	8 43 44		H.
17	I.	Bisection	,,	,,	8 46 44	8 46 0	,,
17	I.	Last contact	,,	,,	8 49 28		,,
17	II. (*l*)	Ecl. R. First seen	,,	,,	9 40 52	9 40 31	,,

Day.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation.	Mean Solar Time of *N.A.*	Observer.
1896.					h m s	h m s	
Apr. 17	II.	Full brightness	Sheepsh. Eqat.	120	9 44 49		H.
17	I.	Tr. Egr. First contact	,,	,,	11 5 6		,,
17	I.	Bisection	,,	,,	11 7 35	11 6 0	,,
17	I.	Last contact	,,	,,	11 10 20		,,
May 11	I.	Ecl. R. First seen	,,	,,	9 48 18	9 48 14	B.
19	II. (*m*)	Ecl. R. First seen	,,	55	9 17 1	9 16 55	D. E.
19	II.	Full brightness	,,	,,	9 18 16		,,

Notes.

(*a*) *Jupiter* very ill-defined. (*b*) Definition of limb not very good. (*c*) Rough. (*d*) Twilight. Probably a little late.
(*e*) Clouds passing, but considered a good observation. (*f*) Cloudy. (*g*) Cloudy. (*h*) Sky hazy. *Jupiter* low down.
(*i*) Cloudy. (*j*) Very cloudy. (*k*) Clouds passing.
(*l*) The first time may be late. The planet was occasionally dimmed by clouds, and the reappearance was suspected about a minute before.
(*m*) Cloudy.

The aperture of the 28-inch equatorial is 28 inches; of the Astrographic Guiding Telescope, 10 inches (referred to as Astrographic Equatorial); of the Sheepshanks Equatorial, 6·7 inches; of the Corbett Telescope, 6·5 inches; of the Transit Circle, 8 inches; of the Altazimuth, 4 inches; of Detached Telescope No. 2, 2·7 inches; and of the Finder of the Astrographic Equatorial, 2 inches.

The initials D., C., L., H., A. C., B., H. F., C. D., R. C., D. E., W. B., J., S. W., and W. S. are those of Mr. Dyson, Mr. Cowell, Mr. Lewis, Mr. Hollis, Mr. Crommelin, Mr. Bryant, Mr. Furner, Mr. Davidson, Mr. Cheeseman, Mr. Edney, Mr. Bowyer, Mr. Johns, Mr. Witchell, and Mr. Stevens respectively.

Royal Observatory, Greenwich:
1897 *January* 8.

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